2005:18

LICENTIATE THESIS

Differentiated Tasks in Mathematics Textbooks

An analysis of the levels of difficulty

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2005:18 | ISSN: 1402-1757 | ISRN: LTU-LIC -- 05/18 -- SE

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To Daniel

Sammanfattning

Syftet med detta arbete är att studera differentieringen i läroböcker för ämnet matematik. Studien är baserad på svenska läroböcker i matematik för år 7 och har utförts med utgångspunkten att alla elever ska få utmaning och stimulans i sitt lärande genom hela grundskolan. Studier och observationer i klassrummet har visat att läroboken har en viktig roll i matematikundervisningen, för både lärare och elever. Det är därför viktigt att studera hur uppgifterna är differentierade i läroboken och hur det kan påverka matematikundervisningen.

Uppgifterna har analyserats ur olika aspekter med avseende på deras svårighetsgrad. Resultatet av studien visar att differentiering sker i läroböckerna, men på en låg svårighetsgrad för alla elever, oberoende av matematikkunskaper. I övrigt visar studien att användningen av bilder inte har någon differentierande roll i de analyserade uppgifterna.

Abstract

The aim of this work is to study differentiation in mathematics textbooks. Based on mathematics textbooks used in Sweden for year 7, the study is performed from the point of view that all students should be challenged and stimulated throughout their learning in compulsory school. Classroom studies and observations have shown textbooks to have a dominant role in mathematics education for both teachers and students. It is therefore important to study how tasks in textbooks are differentiated and how this can affect education in mathematics.

The tasks are analysed with respect to their difficulty levels. The results of the study show that differentiation does occur in the textbooks tasks, but on a low difficulty level for all students regardless of their mathematical abilities. Besides this, the study shows the use of pictures to not have any differentiating role in the analysed tasks.

Acknowledgements

The work resulting in this licentiate thesis was carried out at the Department of Mathematics at Luleå University of Technology (LTU). Financial support was provided by the research school Arena Lärande at LTU and grants were provided by Vetenskapsrådet and the Nordic Graduate School of Mathematics Education for travel and accommodations during various courses and conferences.

First of all, I want to thank my main supervisor Prof. Rudolf Strässer, as well as my assisting supervisors Prof. Barbro Grevholm and Prof. Lars-Erik Persson for their valuable knowledge in the field and support during my progress: with their experiences and personal qualities, all my supervisors have improved and enriched my research. I would also like to thank Dr. Christer Bergsten and Prof. Anna Sierpinska for their suggestions on how to improve the analysis and the presentation of my work. Special thanks are due to my colleagues at the department.

The conversations with and support from Monica Johansson and Christina Sundqvist are highly valued for me. I would also like to thank Wayne Chan for helping me improve my English writing.

Finally, and above all, I would like to thank my family and friends, for putting things into perspective, and especially Daniel, for his love, encouragement and confidence in me!

Luleå, 2005

Anna Brändström

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Introduction

Observations performed in Swedish classrooms (Skolverket, 2003) have shown extensive usage of textbooks in mathematics, and that the existing strands (grouping the tasks according to their difficulty) in these textbooks form the education and organise the students. Studying how these textbooks are constructed and how this affects education in mathematics is therefore important.

My interest in this area originates from my experience as lower secondary school teacher and my growing curiosity of the content in textbooks and their functions. This study is preliminary to a more comprehensive study on textbooks and their implications on differentiation in mathematics education in Sweden.

1.1 Background

In this work, the tasks in mathematics textbooks used in lower secondary school education (year 7) in Sweden are analysed to find out how the textbooks and these tasks are differentiated. Differentiation is often described as a method used to teach in different ways, and give all students the same possibilities to learn. The term is also used when describing organisational differentiation as well as pedagogical differentiation. When differentiating by organisation, education can take place in a whole class, in groups or individually, depending on what is to be taught. Teachers can and should differentiate based on different contents, processes and products (Tomlinson, 2001). According to Tomlinson and Cunningham Eidson (2003), differentiated tasks are important because every student deserves tasks and lessons at his or her level, with respect to knowledge, understanding and skills. A student should be required to think at high level with support from the teacher, and find the work interesting.

The basic material from research documents on differentiation in mathematics education mainly concerns organisational differentiation. Wallby, Carlsson and Nyström (2000) presented an overview of differentiation (pedagogical and organisational) by studying documented research and development work, and concluded that there are reasons to believe that it might not be the organisation of the students that is of importance in mathematical results, but rather the content and structure of education. This makes a pedagogical (and didactical) study of utmost interest.

As described at the beginning of this chapter, textbooks are highly used in mathematics education at lower secondary school. In an overview of research (between 1980 and 1995) on general textbooks in Sweden and their influence, Englund (1999a) presented several conclusions. For teachers, it guarantees the knowledge requirements from the curriculum, gives support when planning and presenting the subject content, and facilitates the evaluation of students. Textbooks give a consistency to the students in their studies and prevent chaos in the classroom by keeping them busy, i.e. the textbook has a very central function in the classroom for both teachers and students. The content of a textbook has educational implications, which for me contributes to the importance of the analysis.

Textbooks and differentiation are not only interesting for Sweden. In an international study, Haggarty and Pepin (2002) studied textbooks and their usage in English, French, and German classrooms in lower secondary schools by analysing textbooks, conducting observations in classrooms and interviewing teachers from the three countries. In the case of differentiation, their results showed that the students of the three countries received different opportunities to learn mathematics. Mathematics textbooks in France stimulated the students with more challenging tasks than those in Germany and England. Unlike Haggarty and Pepin's study, my study does not analyse how textbooks in Sweden differ from any other country. This study presents the current situation in Sweden concerning textbooks and education.

1.2 Objectives

The main objective of this work is to study the issue of differentiated tasks in mathematics textbooks in Sweden. The analysis of the tasks is based on a new tool, and developed as part of this work. The aim of the study is three-fold:

- Describe the structure of a chapter in each of the analysed textbooks to illustrate the strands that separate the tasks in different levels of difficulty

- Construct an analysis tool to study certain important aspects when analysing the difficulty of a task, and apply the tool on existing tasks in the textbooks
- Analyse the tasks and compare the different strands in and between the textbooks based on the aspects in the constructed tool

Furthermore, a general objective is to increase the awareness of how textbooks in mathematics are structured as well as contribute to the development of future textbooks and education.

1.3 Limitations

The limitations of the work concern what aspects are covered by the analysis and what material is analysed. The four aspects used in the constructed framework are 'use of pictures', 'number of operations', use of 'cognitive processes' and the 'level of cognitive demands'. For a more detailed description, see section 4.3.2. An interesting aspect not considered is the use of text, with respect to concepts and the amount of words. This is covered in detail in the discussion.

The material is limited to three textbooks from year 7: Matematikboken X (Undvall, Olofsson, & Forsberg, 2001), Matte Direkt 7 (S. Carlsson, Hake, & Öberg, 2001) and Tetra 7 (L.-G. Carlsson, Ingves, & Öhman, 1998). These textbooks are presented more thoroughly in section 4.1. The analysis is limited to the chapters on fractions; hence, the results do not represent all the chapters in the studied books.

1.4 Results

The results show the three analysed textbooks to have very similar structures. The main parts of the textbooks consist of different strands, grouping the tasks by difficulty levels.

The constructed analysis tool can be used to study the differences between tasks in mathematics. This is done with the four previously mentioned aspects: use of pictures, number of operations, use of cognitive processes and level of required demands.

The aspect 'use of pictures' indicated no differences between the strands, which probably depend on the mathematical content in the analysed chapter, i.e. the chapter on fractions. The other three aspects clearly indicate a difference between the strands. Regardless of the textbook and strand, the tasks are not totally linked to the demands of education. The level of challenge is low in almost all strands, even those intended to be higher. Because of this, an extensive use of these textbooks can result in a low opportunity for students to learn mathematics at their own levels.

1.5 Outline

The theoretical background is further described in chapters 2 and 3. In chapter 2, an overview of differentiated education in Sweden is presented, along with international comparisons, though the text is mostly about the educational system in Sweden. Chapter 3 presents national and international studies on textbooks and tasks, both for general and mathematics education. The chapter ends with a description of what other studies have shown when looking into textbook differentiation.

The methodology is presented in chapter 4. The analysed textbooks are presented and the analysis tool is thoroughly described, both in construction and in use. The results of the textbooks' analysis and the use of the tool are presented in chapter 5. Chapter 6 comprises discussions and conclusions of textbooks' analysis. The quality of the work is discussed, followed by a discussion on the implications of the results. Finally, suggestions are made for further work to present my continuation and give inspiration to others.

2

Differentiation

Differentiation in education is the creation of different learning situations for different students. For example, this can be done by grouping the students in different schools or classes, or by giving them different material to work with. In textbooks, differentiation is connected to content and structure (i.e. tasks in different strands). For me, one goal of education is to develop challenging and engaging tasks for all students, regardless of their abilities and difficulties in the subject. For education to be differentiated, it has to be based and evaluated on the contents taught, processes used and knowledge already received, thereby responding to the needs, interest and readiness of every student. This chapter concerns differentiation in education and the school subject mathematics.

Differentiation is a broad term of the complex process of matching teaching to learning needs and is often described as an occasionally emerging buzzword. In mathematics, it is often discussed whether students should be organised into ability groups or not. The reality is that differentiation has to occur in everyday teaching due to the right of every child to high quality education and individual learning. By using a concept map (see Fig. A.1), Tomlinson (2000) emphasises the three principles of differentiation as respectful tasks, flexible grouping and ongoing assessment and adjustment.

2.1 Learning and teaching mathematics

The subject mathematics is described (Niss, 1994) as a self-supporting pure science that is built up by theorems, definitions and proofs. The subject can be applied to other sciences and practises that makes it interdisciplinary, and is central to many other subjects such as physics. It is built on a system based on different mathematical operations, solving methods and solutions used in mathematical constructions and modelling. Its aesthetic value reflects beauty, joy and engagement for many people who work with it. Education in mathematics is mainly performed by teaching and learning in academic settings. In Sweden, mathematics is one of the main subjects in compulsory school, with Swedish and English being the other two. Niss (1994) also states that the changes in perspectives on learning and knowledge have influenced how people picture mathematics and how it is taught.

2.1.1 Perspectives on learning

Many learning theories exist, but three different traditions are mainly described. The perspectives of a learning situation and the individual‘s role in that situation differ from each other. This text is based on references such as Bransford (2000), Greeno and Collin (1996), Hwang and Nilsson (1996), Runesson (1995), Skolverket (1995) and Säljö (2000).

In the first tradition, learning is described as a transmission of knowledge, e.g. behaviourism and similar theories, and is based on outer behaviours and physical experiences made by the individual that totally diminish the importance of thought and reflection. In a school setting, the student is a passive receiver of the knowledge transmitted by the teacher, and can be described as an empty box to be filled with content in the form of knowledge. Learning can also be seen as the result of the connections between stimuli and response. According to Säljö (2000) textbooks and teaching aids were often based on this model, since students read a paragraph (stimuli) and answered (response). If the answer was correct, the students received positive comments or awards, whereas nothing happened if the answer was incorrect. The role of the teacher in this tradition is to know the subject and properly present it to the student.

The second tradition focuses on the importance of mental processes (thought and reflection) on learning, which is based on a cognitivistic perspective. Here, learning is a result of the student's maturity, i.e. the student's development draws the limit for learning (and teaching). The student is activated by the teacher and is therefore given a central and more active role than in the behaviouristic perspective. Piaget has contributed to this tradition through his development of the stage theory that describes the stages of development for an individual. This was initially not related to learning in school from the beginning, but connections have since then been made. In a text from the Swedish National Agency for Education (Skolverket, 1995) describing what knowledge is, one can find traces of this tradition. Knowledge and experience gained outside school should (as is written) be expanded and deepened. There are also some remarks on the importance of learning new things in the text, though not connected to what is already known (p. 41).

The third tradition has its roots in the work of Vygotskij, and involves learning due to social and technical interplay. Vygotskij worked with something he called the zone of proximal development (ZPD). He assume that the student can attain one point in the learning process by him- or herself. To increase the student's abilities even more, communication with others or the use of a tool is needed. The student learns something for a cause, the problem develops naturally and the solution to the problem is what the student receives as knowledge.

All together, the perspectives of learning have been developed from a traditional understanding that learning occurs passively and isolated to the understanding that it happens actively and jointly. Knowledge has been viewed upon as a package being transmitted between people, and is now viewed upon as being constructed and formed together or with the use of a tool.

I believe that learning happens actively, by using all senses and together with other people. In a school setting, this implies both students and teachers. To me, knowledge is constructed and developed from what you know, together with what you have known and the experiences you make. Knowledge is also developed in connections that make people understand. I believe that this increases when a person tries to describe what he or she has learnt to others or applies it. The student should be the centre of attention in the classroom and the teacher should present new information. Another role of the teacher is to help the student relate to the content.

2.1.2 Knowing mathematics

A historical review on the amount of mathematical information and technological development easily points to the ever increasing and rapidly changing demands on each person. The level of mathematical knowledge needed is therefore higher. Added to calculation skills, there is also a need for critical thinking, expression of thoughts and the ability to solve complex problems (Kilpatrick, Swafford, & Findell, 2001; Kilpatrick & Swafford, 2002; Verschaffel & De Corte, 1996; Bransford et al., 2000). Mathematical knowledge is strongly associated to skills in pure computation. With the help of a piece of paper and a pen, algorithmic calculation in school has decreased to the benefit of mental arithmetic, number sense and abilities on higher levels (Verschaffel & De Corte, 1996; Bransford et al., 2000; Kilpatrick & Swafford, 2002). Therefore, it is not only knowledge about how we best learn that is changing, but also changes in the demands from society that should be and are shaping mathematics education today.

In some international mathematics education studies, one can find many illustrations on what mathematical knowledge really is and its many interpretations. Kilpatrick and Swafford (2002) describe mathematical proficiency with the help of five intertwined strands (Figure 2.1), i.e. what a student needs to be successful in mathematics.

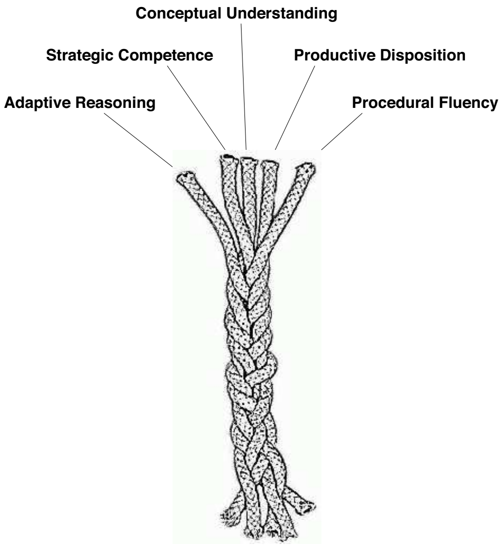


Figure 2.1: Intertwined strands of proficiency (Kilpatrick & Swafford, 2002, p.8)

The formed plait consists of the five strands: understanding mathematical concepts, computing fluently, applying concepts to solve problems, reasoning logically and engaging with mathematics by seeing it as sensible, useful and doable (Kilpatrick & Swafford, 2002).

In the Danish KOM[1] project (Niss, 2003), learning mathematics is paired with mathematical competence. According to the project description, competence consists of knowing and understanding, doing and using and having a well-founded opinion of it. In the project, two groups with eight competencies for mathematics are identified. The first includes competencies needed to ask and answer questions, i.e. mathematical thinking, formulating and solving problems, building and analysing mathematical models and being able to follow and use reasoning. In the second group, abilities involving knowing and

[1]Initiated by the Danish Ministry of Education and other official bodies

using mathematical language and tools are described, i.e. making connections between representations, being able to communicate through mathematics and using and relating to the tools and helping aids.

Verschaffel and De Corte (1996) describe the mathematical competence needed in terms of arithmetic. They state that arithmetical knowledge is more than rules, solving methods and applications. The needs required to develop knowledge at a higher level are described as: discover, reason, reflect and communicate. They also express the need for every student to develop a positive attitude towards mathematics, to look at it as a tool and to know his or her own mathematical ability.

The Programme for International Student Assessment (PISA)[2] consists of a study performed every third year (with the first done in 2000) of how well prepared 15 year-old students are for any future challenges they are to meet[3]. Instead of studying how much the students have learnt in a specific area, the focus lies on assessing how well they can use their knowledge in reading, mathematics and science. Their mathematical literacy was measured in the first study through the following model:

1. Recognise and interpret problems they meet in their daily lives
2. Transform the problem into a mathematical context
3. Use their knowledge of mathematics to solve the problem
4. Reflect on the result by looking at the information given in the beginning
5. Reflect on the chosen and used methods
6. Formulate and present the solutions

Mathematical competencies are one of the major aspects in the framework (OECD, 1999). The skills studied include elements such as mathematical thinking and argumentation, modelling, representation, communication, problem posing (and solving) and aids and tools. In the framework, the skills are arranged in three classes of competency (OECD, 1999, p.43):

- Class 1: Reproductions, definitions, and computations
- Class 2: Connections and integrations for problem solving

[2]For more information, see http://www.pisa.oecd.org
[3]Sweden is one of the 32 participating countries

- Class 3: Mathematical thinking, generalisation and insight

The definition of mathematical literacy (used by OECD/PISA) does not use these three classes to form a hierarchy, making the tasks in Class 3 more difficult than Class 2 or Class 1. Instead, OECD/PISA gives importance to students "demonstrating the capacity to perform tasks requiring skills in all three competency classes" (OECD, 1999, p.44).

The terms competency, literacy and proficiency are very similar when describing the different frameworks used, and ultimately describe what a student should learn and train during extensive schooling. The three classes of competencies described in PISA seem to occur in all the other descriptions as well. The students are not only required to know the mathematical definitions and calculations ('pure' mathematics to some), but also be able to use the knowledge in different surroundings and different tasks (not only those in the textbooks), and reflect, reason and present their choice of methods and results.

2.1.3 Education in Sweden

In Sweden, the Education Act regulates the Swedish school system. All schools follow national goals and guidelines as presented in the curriculum and national assessments. All schools have their own local profile and school plan[4].

The educational goals in Sweden are two-fold (Skolverket, 1997). Social, economical and technical development is needed for society. The individual needs to understand and be active in, for example, democratic processes as well as get the aesthetic values out of the surrounding world. Education should give opportunities for learning in compulsory school, upper secondary school and throughout a lifetime.

The syllabus (National Agency for Education, 2000) is one of the documents that, together with the curriculum, controls and guides education in Sweden. Each subject taught has a text describing the goals to achieve, the goals to strive for and the criteria for assessment.

When dealing with mathematical knowledge, seven parts are emphasised by the syllabus (Skolverket, 1997, p.13-21):

1. Mathematical confidence is perhaps most important when learning mathematics. Having no confidence can change a person's life and future in

[4]For more information on the Swedish school system, see the printing from the National Agency for Education (2004), giving a general description of the educational organisation in Sweden.

many ways. When dealing with mathematics, confidence in the subject affects decisions in education, work and democratic settings

2. Historical connections are needed to understand the mathematical developments in the surrounding environment

3. Comprehensive concepts and methods are the basic parts of the curriculum goals. This is what mathematics is based on

4. Possessing a mathematical language requires thinking and reasoning. From text and pictures a transformation into mathematical symbols and figures is needed, and vice versa. An understanding is developed through reasoning as opposed to simply offering the correct answer

5. Problem solving is often presented in textbooks as already drafted tasks with given numbers and sometimes given methods in the introduction. Often, the tasks are presented in such a way that the students do not understand the intended real-life situations. In the "real world", problems have to be formulated, number and figures have to be collected, the solution method has to be chosen and the answer has to be analysed to make it realistic

6. Modelling is characteristic for mathematics, and especially applied mathematics. It can be described as schemes or thoughts on how to analyse reality or theory. Models are used in many areas outside the classroom and textbook tasks, e.g. calculating the speed of a car, making statistical surveys or calculating bacterial growth

7. The use of technological tools, such as the calculator and computer, are nowadays highly necessary and accepted in society. The tools should not totally diminish the use of algorithmic calculation, since it is necessary in the knowledge, understanding and skills of mathematics

Besides the aims in the Swedish curriculum, a very concrete goal in education is for every student to pass the subjects English, Mathematics and Swedish, which is one of the requirements for entering upper secondary school. The alternative is a special program in upper secondary school adapted for the special needs of the student.

2.2 Teaching according to the needs

There are three common ways to look at learners and their needs:

- Common needs - Everyone is the same
- Distinct needs - Some people are similar
- Individual needs - Everyone is different and unique

Students are often considered having common needs, i.e. students have the same capacity, ability and potential to learn and at the same speed (O'Brien & Guiney, 2001). According to Hart (1996), the question of teaching for different abilities was raised in England after surveys done in 1970 and 1980 showed that teaching tended, independent of grouping, to aim at the average student. There were simply no challenges for students above or below this level. There also seemed to often be a problem in defining the ability of the average students. This implied that teachers had low expectations and a too narrow approach in their teaching. The teaching material was described as being over-directive and reduced the opportunity for students to think for themselves, since they were supposed to work without any reference to the teacher.

2.2.1 Differentiated instruction

Upon reflection, education has not always been directed to all groups of society. Some subjects have a tendency of elitist thinking, to educate a small group of people so as to have top students. The concept of mathematics for all describes the vision of teaching and learning mathematics independent of ability and future plans, or as mathematics for all students at all levels. Not all students aim to become mathematical experts, but they should at least have the opportunity. Allexsaht-Snider and Hart (2001) state that to achieve the goals of a school for all, knowledge about diverse learners, classroom processes and teaching practises should be emphasised.

The mathematics teacher has the task to meet the needs of all individuals in his or her care. Many researchers state that grouping students by their ability is the solution to the task, but the problem of instruction remains. According to Haggarty (2002) and the Swedish National Agency for Education (Skolverket, 2003), a common scene in the classroom is the introduction by the teacher followed by student practice, to which the most time is devoted.

From the teacher's perspective, the introduction has to be adapted to the students in the group or class. It has to start and end somewhere, cover a specific topic and be adjusted to the working speed of the students. Other things to consider are the practice performed by the students and the amount or difficulty of the material or activity. The differences between the students can vary and be educational, psychological, physical, social, socio-economical and cultural (Haggarty, 2002).

Tomlinson and Cunningham Eidson (2003) refer to education as having two aims: emphasising the needs of each student and maximising his or her learning capacity. For differentiation, teachers can act upon five elements:

1. Content
2. Process
3. Products
4. Affect
5. Learning environments

These elements are also described in Tomlinson's concept map (Appendix A) from her earlier work (Tomlinson & Demirsky Allan, 2000).

Content is often obtained from many different sources. National, state and local standards, or curricula provide the framework for what to teach. The local curriculum guide (constructed in schools) and textbooks further define the content. However, the main source of content is the teacher, based on his or her knowledge of the subject, and the students. The teaching methods and materials used are decided by the teacher and give students access to the content. Demonstrations by the teacher and the usage of textbooks, supplementary materials, technology and excursions are all different ways to differentiate by using content. The process begins when the student stops being a consumer and starts producing.

In this sense, products are what the students demonstrate as the knowledge they possess, i.e. what they have come to know. Based on the needs of the students and their grades, product assignments should call on students to use what they have learned, should be clear, give a challenge and have specific criteria for success.

All students need to feel that they belong to the group and are important to it. They also need to feel challenged and know that they have the opportunity to achieve at a high-level of expectation. In a differentiated classroom, the teacher has to adapt to the student's knowledge, skills and understanding.

Tomlinson and her colleagues (Tomlinson & Demirsky Allan, 2000; Tomlinson & Cunningham Eidson, 2003) describe the need for a flexible learning environment as a hallmark of a differentiated classroom. This is possible when using space, time and materials in a variety of ways as well as including the students in the decisions (Tomlinson & Cunningham Eidson, 2003).

Haggarty (2002, p.194) describes four ways of how differentiation can be done inside the classroom:

- Outcome
- Rate of progress
- Enrichment
- Setting different tasks

When differentiating by *outcome*, all students are given open-ended tasks. Their responses to the questions are at different levels, thereby illustrating their differences in ability. *Rate of progress* is often referred to as the acceleration of high achievers. The student works through the course at his or her speed, also called individualisation by speed (Wallby et al., 2000). Supplementary tasks are given to students to broaden or deepen their skills, i.e. differentiation by *enrichment*. High attaining students are often given these kinds of tasks, to keep the class together and work on the same topic. Because of the rate of progress, it is rare that students with low attainment are faced with these tasks. When differentiating by *setting different tasks*, the students do not work with the same material from the start. When planning a teaching unit, it is therefore important to know what the student knows to be able to give him or her suitable tasks.

2.2.2 Educational settings

In Marklund (1985) and Wallby, Carlsson and Nyström (2000), differentiation is described as either organisational or pedagogical. In O'Brien and Guiney (2001), the organisation of students is tackled as accommodation not differentiation, i.e. that organisational questions should not be discussed as educational differentiation. The question of how to define the different situations is very complex and the different types of groupings and methods used can be difficult to arrange in different descriptions.

Ability or mixed ability grouping?

The question of how to group students in mathematical education has been discussed in many countries for a long time.

In England, historical and political developments have greatly influenced teaching. In the 1960s, 96% of the schools were grouped by ability (known as streaming). The negative effects of this were that teachers underestimated working class children and low-streamed groups were given less experienced and less qualified teachers. All forms of ability grouping were excluded in 1967 (as a recommendation); there was instead (political) support for grouping by mixed ability. In the 1990s, many schools returned to policies of ability grouping. The attention had then been turned away from equality in education and towards academic success for the most able. In 1993, the government directed schools to group students by ability, though many studies showed the negative effects of doing this (Boaler, 1997b, 1997c).

In Sweden, ability grouping has been used in education for a very long time. After the lengthy involvement of projects that studied specific grouping situations (Wallby et al., 2000), the curriculum development removed the instruction of grouping students according to ability in 1980.

Boaler (1997c, 1997a, 1997b) is one researcher who has studied grouping in England. In one of her studies, two schools are the objects of research. The first school used ability grouping; the second used mixed ability grouping. The result showed that students in groups of mixed ability achieved higher than those in ability groups. According to Boaler (1997b), success in mathematics is not dependent on students being able or if they work hard. Instead, working quickly, adapting to the norms of the class and thriving on the competition made up the picture of success.

When studying the students in the 'top set', Boaler(1997c) concluded that the lessons were of fast pace since the teacher introduced the subject very fast and the content was to be finished quickly, the top set consisted of students of mixed abilities who waited for each other, and a pressure to succeed in the set resulted in competition between the students. Boys were often more willing to play by the rules and perform without requiring any meaning to what they had learnt, whereas girls suffered from the working speed and competition in the class.

A study was done in six other schools as a follow up. The result showed that teachers working with ability grouped students often used one student as a model and based the teaching on the textbook (Boaler, Wiliam, & Brown, 2000; Boaler & Wiliam, 2001). In mixed ability groups, the teachers let the students work at "their own pace through differentiated books or worksheets" (p.91), though the students were unsatisfied with their group arrangement.

Ireson and her colleagues (Ireson, Hallam, Hack, Clark, & Plewis, 2002; Ireson, Clark, & Hallam, 2002) have also studied ability grouping in secondary schools, illustrating that students attaining higher in grade six performed better in ability groups, whereas pupils attaining lower made more progress in mixed ability groups.

In a research review, Harlen and Malcolm (1999) studied research on setting and streaming. In secondary school studies, many of the results were contradictory, based on method used, how the groups were formed and the attitudes of the teacher. This led to the conclusion that flexible within-class grouping should be adopted for the student to get more interaction with and support from the teacher and other students. Whichever grouping is done, Harlen and Malcolm emphasise on the importance of meeting the student's needs by providing challenge and support.

2.2.3 Differentiation in Sweden

The decision for a comprehensive school in Sweden was taken in 1962, after a long period of try-outs on how to organise education. The Swedish school became no longer a school of selection but a school of choices. In years 7 and 8, some subjects had alternative courses, while different programs could be chosen in year 9. When changing from the old school system, this arrangement was done because many were sceptical of waiting until the 9th year before differentiating according to ability. The main subject of scepticism was the teaching of gifted students. The opinion was that their education would be negatively affected in at least three aspects: gifted students had the right to advance more rapidly and go deeper into the subject matter, there was a risk that gifted students would become bored by working at a slower pace, resulting in them getting no satisfaction out from their work and gifted students would take a risk in not getting the knowledge they need in their higher education (Husén, 1962, p.56).

When discussing the less gifted students, the comparison between different students could have a negative effect. Less gifted students could be marked as stupid, decreasing their level of self-confidence.

IMU project

According to Olsson (1973), a pilot study started during the school year 1959/60. The lower secondary school had to organise the students (in year 8) in joint classes, while the students were divided in alternative courses in other subjects. Problems occurred due to difficulties when using a textbook, it was not possible to teach a mixed ability class with only one textbook and several books were needed. Students working at the more difficult level were therefore using self-instructive material (e.g. material from a distance course). The teacher felt that individualisation was achieved and the School Commission did not react negatively to the solution. After several years, a comparison between the two solutions (individualised and whole class teaching) was needed. The school became an experimental school and studied the following questions (Olsson, 1973, p.11, my translation):

- Can it be possible to teach a class of 20-30 students individually?
- What teaching means are needed and how should these be designed?
- What working load do students and teachers get?
- What will be shown concerning students attitude, standard of knowledge and will they achieve higher independency and responsibility?

The experiment included two different cases. In the first case, teaching was performed as usual, with the ordinary textbook and alternative courses. The students worked as one group, both in receiving instructions and working pace. In the second case, material from the distance course was mixed with the original textbook. Students gained individual instructions and worked in their own pace.

To meet the demands of teaching in mixed ability classes (as in the pilot study), and to solve the problem of shortage of teachers in mathematics, the experiment was developed, and in 1964, the IMU-project started. IMU stands for individualised mathematics education, with the following goals described in four points (Olsson, 1973, p.21):

- Construct and test a self-instructing student material in mathematics
- Test suitable teaching methods when using this material
- Test how students should be grouped and teachers used, in order to get a maximal effect of material and methods

- With the help of the constructed material, measure the effects of the individualised teaching (eventually together with comparison to a conventional teaching of a class)

The students were given self-instructive material, and methods and solutions were found in the textbook. The role of the teacher was supposed to be less important, but the result of the project showed otherwise.

Results of IMU?

The IMU-material was developed during the project. As a result of the project, improved teaching material was also developed to the better, though it could never replace the teacher in the form of a self-instructing material (Wallby et al., 2000). Another result after the project was shown in textbooks used in Sweden, which had the same structure as the material used in the study (Marklund, 1973, p.173):

> The set of textbooks used for mathematics grades 7-9 has been strongly influenced by IMU. In today's situation, very little or no Swedish educational material in mathematics for compulsory school has not been affected by IMU. [My translation]

In my review of mathematics textbooks (Brändström, 2002), some findings imply that this is still the case in textbooks used for mathematics today (e.g. 2005). The review done in the essay is continued with this work, focusing on differentiated tasks.

3

Textbooks and tasks

Textbooks generally have three main functions: present the content to be taught, define the goals and teach the discipline (Svingby, 1982; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Teachers consider the textbook as being very practical to use and as legitimising the content taught (Hellström, 1987; Chambliss & Calfee, 1998; Englund, 1999b; Juhlin Svensson, 2000). Englund (1999b, 1999a) and Gustafsson (1982) discuss the often-used assumption that textbooks control education, while Englund (1999b) emphasises the importance of the textbook in education, since people treat it either knowingly or not as something fundamental. This has been questioned in Sweden, where a recent report (Skolverket, 2003) contains the following quotation describing the view of mathematics in the classrooms:

> Mathematics is, for teachers and students, simply what is written in the textbook. (p.39, my translation)

According to Henningsen and Stein (1997) mathematical tasks are central to the students' learning because they "convey messages about what mathematics is and what doing mathematics entails" (p.525), a very comparable quote to the one above. Gilbert (1989) emphasises the importance of studying the textbook content and structure with its use in the classroom by teachers and students, i.e. the researcher could interpret the textbook's content differently in its natural environment. I initially intend to analyse three textbooks and discuss their content. The investigation of their usage will be left out in this work and hopefully be undertaken later on.

In this theoretical chapter, previous studies on mathematics textbooks in general and on differentiation in textbooks in particular are presented to describe what has been going on until now. In the final part, some theories on how one can study mathematical activity are presented.

3.1 Content and structure

The content, structure and organisation of mathematics teaching were guided by the textbook, according to observations from Swedish classrooms in 2002 performed by the National Agency for Education (Skolverket, 2003). With regards to differentiation, students were grouped based on levels in the textbooks. If the textbook is strictly used as a guideline (describing what is important to learn and how to best learn it), it is critical to look into what the textbooks really offer.

Studies on textbooks can have different foci. Pepin and Haggarty (2001) reviewed relevant literature in the area, showing that mathematics textbooks have so far been analysed with four different foci. These are studies with mathematical or pedagogical intentions, sociological contexts and representations of cultural traditions.

3.1.1 Mathematical intentions

When dealing with mathematical intentions one can examine how mathematics is represented, implicit beliefs on the nature of mathematics and the presentation of mathematical knowledge. Scientific and school knowledge are discussed as two distinct fields (Pepin & Haggarty, 2001). Research communities generally accept scientific knowledge as 'real' mathematical knowledge; school knowledge is therefore knowledge presented through textbooks. Love and Pimm (1996) express the two as "versions of mathematics for particular purposes" (p.375).

As an example, Friberg and Lundberg (2003) analyse the geometry presented in mathematical literature used at an upper secondary level. Two textbooks were analysed in the study with a focus on content structure. According to the study, the content of the textbooks was mainly tasks of algebraic nature, though none focused on the historical part of geometry.

Bremler (2003) studies how the mathematical concept of differentiation was introduced in Swedish textbooks published from 1967 to 2002. He found that proof of the differentiation formula was not presented in any textbooks published after 1994 and that the purpose of learning derivative was rarely described during the period studied.

Pepin and Haggarty (2002) study the topic of angles in textbooks from England, France and Germany. To assist them, an analysing schedule focusing on the authority of the text, the author's views of mathematics, analysis on content knowledge and pedagogical intentions among other things were used. Each country had a different focus: in France, how the low achievers might cope with the demands put on them; in Germany, how differentiation was achieved between the different types of school; and in England, how to increase the learning opportunities for all students. The study showed that mathematics in England appeared to be “a set of unrelated but utilitarian rules and facts” (p.142). Words were rarely used in the textbooks and it seemed as if students in England were to learn mathematics solely by repeating exercises, “Mathematics was there to *be done*” (p.142).

Another study is presented by Törnroos (2001), who analyses the nature of the intended and implemented curriculum in Finnish textbooks. The Third International Mathematics and Science Study (TIMSS) and its curriculum study influenced his analysis of textbooks. His questions concerned the content of textbooks in school grades five to seven and the differences between them. The result showed that one could find less new content in textbooks for grades five and six and more new textbook content for grade seven. Certain content areas (percentages and probability) were widely covered in grade six compared to grade seven.

3.1.2 Pedagogical intentions

Literature on pedagogical intentions attempts to point out three themes: examining how textbooks help the learner within the content of the text, within the methods included in the text and by the rhetorical voice of the text (Pepin & Haggarty, 2001).

When studying the link between the curriculum and mathematics textbooks in Sweden, Johansson (2003) is partly inspired by the Third International Mathematics and Science Study (TIMSS) and its curriculum study. By doing a content analysis of the same textbook for lower secondary school from three periods (with three different curricula), she studies the textbook as the potentially implemented curriculum. The results show that the content in the different textbooks were similar, though the textbooks do not present the same image as the curriculum.

In Brändström (2002) six textbooks used in grade 7 of Swedish lower secondary schools are analysed. The intentions are more pedagogical than mathematical. The structure, content and layout of the textbooks are studied by looking at methods to organise the students, present subject areas and use pictures. The results showed that most textbooks grouped students into ability levels through tasks, subject areas of each book were the same and presented in the same order and the illustrations were modern prints and pictures of interest to students (for example, a hamburger).

Areskoug and Grevholm (1987) performed a similar review of textbooks used in Swedish classrooms in 1987. The aim was to analyse three major themes: textual content, work procedure and methodical structure. The results they emphasised were a lack of identification in the tasks for all students, a lack of alternative activities and material for teachers to use or read and a lack of guidance of how teachers should choose teaching units and contents from the large amount of material presented in the textbooks.

3.1.3 Sociological contexts

Dowling (1998) emphasises the use of sociological strategies to evaluate textbooks. Sociological analyses of textbooks include that of gender, ethnicity and class, as well as ideology. According to Dowling, attention to the sociology of education has rarely been directed at mathematics education. Social considerations have tended to be placed in the background and categories such as ability, achievement and needs in the foreground. He believes that ability, achievement or needs do not exist, and instead argues for variables composed in and by the practices of schooling that do not measure the students' qualities as students.

In his own work, Dowling (1996) analyses the sociological texts of textbooks used in the UK. The textbooks were meant for students with different ability levels; therefore, he selected two textbooks, one designed for low achieving students and one for high. His result showed differences in the textbooks regarding content, treatment of topics and expectations and aspirations of the students who were supposed to use the textbooks.

3.1.4 Cultural traditions

In Pepin and Haggarty (2001) a text is described as not only delivering systems or facts, but also results of political and cultural activities. Their study not only presents the reflected system of ideas and beliefs in the textbooks, but also the whole process in the classroom.

Pepin and Haggarty (2001, 2002) analysed textbooks use in English, French and German classrooms at lower secondary school to see the cultural influences. The whole study was based on analysed textbooks, interviewed teachers and observations in the classroom. The evolving pedagogical principles of the teachers and the systems' educational and cultural tradition were, according to the study, shaping the classroom culture. A finding in the study implied that students in England with "intermediate" knowledge were never challenged with difficult tasks, since the teachers assumed the students were not able to solve problems above their specific level of ability (Pepin & Haggarty, 2001).

Törnroos (2001) and Johansson (2003) are both studying the connection between curriculum and textbook with the help of TIMSS curriculum study. Their studies can be referred to as cultural because of their connections to each country's curriculum, which is shaped by historical and political influences.

3.2 Tasks in education

An educational task can be defined in many ways. When analysing a task's working scheme, Stein and Smith (1998) use the following definition:

> [...]a segment of classroom activity that is devoted to the development of a particular mathematical idea. A task can involve several related problems or extended work, up to an entire class period, on a single complex problem. (p.269)

Niss (2003) defines it as an oriented activity, where the actions are oriented towards, for example orders or challenges. It can be formulated orally or in writing by a person or a group using terms such as: compute... , solve... , prove... . A task can also consist of questions like: how many... ?, what is the relation... ?, etc. According to Niss, the task forms "the center piece of attention and activity" (p.17) in a classroom.

The mission of the task is to solve it and find an answer. Niss describes tasks as possibly being from different categories such as questionnaires, exercises and problems. He defines the three as follows (p.20-21):

- A questionnaire: A collection of tasks concerning facts such as definitions or results of computations
- Exercise: A task of primarily routine type or operations in straightforward combinations
- Problem: A task of non-routine type with considerations of operations

Exercise and problem are not absolute concepts since many people are defining exercises as problems and vice versa.

In using the word 'task', I have included exercises, problems and word problems. In this case, all analysed exercises in the textbooks are defined as tasks.

Tasks are often designed to reveal the facts the student knows (or not), the techniques they can master and if the techniques can be used in certain situations, where all are related to the underlying curriculum and its goals for education. Consequently, tasks used in assessments point out what "the essential components of mathematics and mathematical ability are considered to be" (Niss, 1993, p.20). This can be connected to the quotation at the beginning of this chapter that "mathematics is simply what is written in the textbook", i. e. mathematics is a wider description, including theory, examples and tasks presented in the textbooks.

Henningsen and Stein (1997) present a conceptual framework (Fig.3.1) based on the construction of mathematical tasks used for this kind of study. They define a mathematical task as a classroom activity whose purpose is to focus the students' attention to a specific concept, idea or skill.

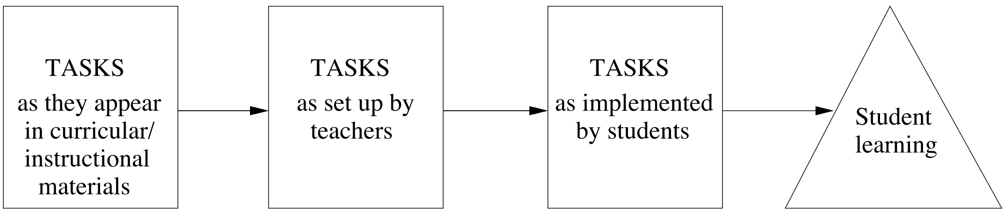


Figure 3.1: The mathematics tasks framework (Henningsen & Stein, 1997, p.529)

In the framework, tasks pass through three phases. The first phase is the task's appearance in curricular or instructional materials as task developers write them, the next phase is their use by teachers and the final phase is the implementation by the students in the classroom. All three phases are part of mathematics education and the learning process of the student. Two dimensions, task features and cognitive demands, are added to these phases. The first refers to important aspects identified by teachers; the second refers to the thinking process required to solve the tasks and use of the process by the student in the actual implementation phase. The first phase is of interest to this study. The other two parts will be studied later on.

3.3 Taxonomies and frameworks

To classify how tasks are differentiated, a framework is needed to analyse the difference between them. Taxonomies are classification schemes according to a predetermined system, and their use in education provides a basis for information retrieval, analysis and discussion as well as increasing the accountability and the quality of the study.

The results provide a conceptual framework for discussion, analysis or information retrieval. The best-known and used taxonomy in education was developed by Bloom (1956) and focused on objectives assessment (see section 3.3.2.

Does a tool to analyse tasks and their differences in textbooks exist? Some of the frameworks presented here are mainly used when studying educational activities, not only mathematical and in the textbooks. I will clarify the connections by presenting how the taxonomies and frameworks are viewed upon and used in educational research.

3.3.1 The SOLO taxonomy

Biggs and Collis (1982) developed the Structure of the Observed Learning Outcome (SOLO taxonomy) to classify the response of students to mathematical tasks. The taxonomy studies the quality of learning by using a hierarchical model. Stage theorists (such as Piaget) often use hierarchical models where three stages (pre-operational, concrete and formal) are followed by each other. According to Biggs and Collis, the student can be labelled into one of these stages and carries the given label until the next stage is reached. Therefore, it is unnecessary to instruct the student at a higher level than where he or she is currently.

Biggs and Collis use five stages in their taxonomy: prestructural, unistructural, multistructural, relational and extended abstract. A description of how to use the taxonomy in different educational subjects is given. Elementary mathematics can be evaluated in the following way (Biggs & Collis, 1982, p.61-93)):

- Prestructural: Here, evaluation is difficult and analysis is irrelevant. The student has not reached a sufficiently high cognitive level.

- Unistructural: At this level, working memory capacity is low. The responses include arithmetical items that involve making one closure.

- Multistructural: The student's response shows facility with large numbers involving single operations and a number of operations in sequence, when the numbers are kept small. The student cannot make connections between parts in the task.

- Relational: The student shows (in his or her response) an ability to connect different parts of the tasks in relation to the whole.

- Extended abstract: The student's response shows an ability to consider the possibility of more than one answer to any item. Connections can be made beyond the given subject area.

Biggs and Collis state that the SOLO taxonomy is a criterion-referenced measure of the quality of learning, i.e. the evaluation shows a close relation between evaluation and instruction. Connections to taking a driver's license test are made: since certain standards are to be met, one either meets them or not. After failing the test, further instructions are given and the person retakes the test. Connecting this to the taxonomy would imply that a student who has not reached the higher stage would receive more help to reach the stage the next time.

In Biggs and Collis (1982), the task of the learner is described as twofold:

> First he has to learn some data, such as facts, skills, concepts, or problem-solving strategies. Second he has to use those skills, facts, or concepts in some way, such as explaining what he has learned, or solving a problem, or carrying out a task, or making a judgement (p.3).

Evaluating the learner's knowledge can be done either quantitatively or qualitatively, though in mathematics education, quantitative methods are used most. One example is the final test, where the number of correct answers (given by a score of points) decides what grade the student receives.

When discussing how to teach, Biggs and Collis emphasise that the teacher must be engaged in individual diagnostic teaching. The evaluation is not done according to the taxonomy and its theory if the test results from one student are related to the class-average score.

If the student is challenged with too difficult tasks, they will only try to learn and remember the mathematics formulas instead of understanding them.

3.3.2 Bloom's taxonomy

By assuming that ability can be measured along a continuum from plain and simple to rather complex, a framework developed during the 1950s has been extensively used. Benjamin Bloom (1956) and a group of educational psychologists created this taxonomy to represent the intended outcome of the educational process and categorises the students' behaviour (Bloom, 1956). Krathwohl (2001) describes this taxonomy as "a framework for classifying statements of what we expect or intend students to learn as a result of instruction" (p.212). Its creation was the result of many years of needing a tool to measure achievement among students (Kilpatrick, 1993), as well as classify intended activities and outcomes of students and offer a structure in which one could group test questions. According to Kilpatrick (1993), the taxonomy attempts to reflect the differences of the student's actions (made by the teachers), be logically and internally reliable, reflect the psychology of learning by discussing how to teach and learn, be both neutral and complete and be suitable in all school subjects. Bloom realised that the taxonomy could, besides being a tool for measurement, serve as a common language for learning goals, a means to determine the similarity among the different objectives and activities and a view of the variety of educational opportunities (Krathwohl, 2001).

Three domains of educational activities are identified in the taxonomy: cognitive, affective and psychomotor, as described by Bloom (Bloom, 1956) in the following way. The 'cognitive' is demonstrated by knowledge recall and intellectual skills (e.g. comprehending ideas and applying knowledge). The 'affective' is demonstrated by behaviour in a learning situation (e.g. awareness, interest and ability to listen), whereas the 'psychomotor' is verified by physical skills (e.g. coordination, strength and speed). Bloom et al. (1956) describe the cognitive domain as central to the work of test developers and emphasise that the definitions of objectives are more clearly described there than in the other two domains. Because of this, it has been more useful and has received more attention during recent years (Kilpatrick, 1993).

The cognitive domain is arranged into six hierarchical categories, beginning from the simple behaviour and building to the most complex: knowledge, comprehension, application, analysis, synthesis and evaluation. The categories can be thought of as levels of difficulty, where the first category has to be mastered before advancing to the next. A student that performs at a higher level demonstrates a more complex level of cognitive thinking (Bloom, 1956).

My interpretation of the taxonomy is presented below (Table 3.1), where the information is mainly taken from the original description written by Bloom et al. (1956).

LEVELS	DEFINITIONS	VERBS
KNOWLEDGE	Remembering or recalling appropriate, previously learned information in the form of specifics, methods, structures or settings.	Write, list, label, name, state, define
COMPREHENSION	Understanding and apprehending of what is communicated and can make use of the material or idea. This without relating to other material or seeing its complete connections.	Explain, summarise, paraphrase, describe, illustrate
APPLICATION	Applying previously learned information (for example general methods and technical principles) in new and concrete situations to solve problems that have single or best answers.	Use, compute, solve, demonstrate, apply, construct
ANALYSIS	Breaking down information into component parts, clarifying the structure and procedures (i.e. the hierarchy of ideas and their relations to each other).	Analyse, categorise, compare, contrast, separate
SYNTHESIS	Combining prior knowledge and skills to produce a pattern or structure (a whole) that was not clearly there before.	Create, design, hypothesise, invent, develop
EVALUATION	Judging the value and satisfaction of material and methods. The judgements are based on criteria given or determined by the student.	Judge, recommend, critique, justify

Table 3.1: The original Bloom's taxonomy

The difficulties that arise when using the taxonomy are related to the problem of hierarchical levels and the assumption that all learning outcomes can be placed in one of the specific levels. The difficulty in interpreting the categories, the independence of content from process and the fact that the categories are isolated from any context, have also been criticised (Kilpatrick, 1993; Gierl, 1997; Brown, 2004). Much of the criticism has been on the time consumed, since user creates the relations to the educated subject (Kilpatrick, 1993). This is because the categories in the taxonomy are given as a set of tools and do not present how they can be used to design a good educational environment.

Gierl (1997) compares the cognitive representations among test developers and students by using a mathematical test and Bloom's taxonomy. The study determines if the taxonomy provides test item writers with an accurate enough model to predict the cognitive processes used by elementary school students in solving tasks. She assesses the three objectives knowledge, comprehension and application. The results showed that the cognitive domain in Bloom's taxonomy does not provide test writers with an accurate model. The expected processes given by the test writers matched the students' processes by 54%, though the results were better for students with a high mathematical achievement. According to Gierl, the objectives were too general and many of the cognitive processes were not made visible in the analysis.

Despite the criticism, the view of mental abilities (arranged in a linear hierarchical system along with mathematical thinking and achievement) has influenced the 20th century assessment practice (Kilpatrick, 1993). Brown (2004) describes how teachers can use the taxonomy, here are some of the suggestions (p.14):

- Use Bloom's for preparing the questions you will ask the students in the seminar
- Check that some of the questions were of higher orders - and that the students' responses were too
- Use the taxonomy to check the types of questions you are setting in assignments and examinations

According to Kilpatrick (1993), several frameworks are based on parts of Bloom's taxonomy for the construction and analysis of mathematical achievement tests. The taxonomy has also been adapted into a planning tool for classroom use. Similar discussions are presented in other frameworks when it comes to classifying the cognitive processes, but none has been used as widely as Bloom's taxonomy (Gierl, 1997). According to Krathwohl (2001), the taxonomy has mainly been used to classify curricular objectives and test tasks, and to show their breadth (or lack thereof) across the spectrum of categories. The main results of these analyses illustrated a heavy emphasis on objectives requiring recognition or recall of information. Important goals are seen as the categories from comprehension to synthesis.

During the 1990s, Anderson (2001) and her colleagues revised the taxonomy for use in the twenty-first century. Some improvements were made including important changes in terminology, structure and emphasis (Table 3.2).

Bloom's original taxonomy	Anderson's revised taxonomy
Knowledge	Remembering
Comprehension	Understanding
Application	Applying
Analysis	Analysing
Synthesis	Evaluating
Evaluation	Creating

Table 3.2: Bloom's taxonomy, revised by Anderson

When describing terminology, the number of categories was unchanged. Three categories were renamed and two were interchanged. All categories were also changed from nouns to verbs (see Table 3.2), since the taxonomy reflects the thinking among students. Thinking is an activity, i.e. an active process, easier connected to a verb (that describes an action). The category 'knowledge' was renamed to 'remembering', since knowledge is a product of thinking and not a form of it. 'Comprehension' was changed to 'understanding' and 'synthesis' was changed to 'creating' to reflect what was defined inside each category. The structure was changed to a two-dimensional table (see Appendix B), and the dimension on the various forms of knowledge (products of thinking) listed as factual, conceptual, procedural and meta-cognitive were added. The two last categories were interchanged because the old structure had been criticised for not ordering the categories in terms of increased complexity. This has been discussed and questioned, since one can be critical without being creative, while creative thinking often requires critical thinking. The revised version, whose aim is for broader audience and offers more detailed descriptions of the sub-categories, focuses primarily on the use of the taxonomy.

Williams (2002) uses a similar framework to evaluate a task and its potential to stimulate creative thinking. He connects the six levels in Bloom's revised taxonomy with the processes of abstraction (recognising, building-with and constructing) described by Dreyfus, Hershkowitz and Schwarz (2001). This was useful when comparing the possible and actual student response to mathematical tasks. According to Williams (2002), it should increase the awareness of the teachers to complex cognitive activities related to creative thinking among the students.

I consider the advantage of this framework to be its use in analysing mathematical test tasks and it should therefore be useful when studying textbook tasks. Are there any differences among differentiated tasks when examining the cognitive processes used? Parts of the revised version of Bloom's taxonomy will be used in this study, since it seems easier to use and more up-to-date. A detailed description of how the framework is used can be found in the methods chapter (see section 4).

3.3.3 The QUASAR project

To study what makes a difference in how students view mathematics and what they ultimately learn a project was started among a group of researchers (Smith & Stein, 1998). The project called Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) was a five-year study (1990-1995) of mathematics education reform in urban middle schools in the United States.

The aim was, among others, to describe key features of good instructional programs and present them to teachers and educators, while spreading the already existing good examples (Silver & Lane, 1993).

The project rested on the premises that it was both necessary and possible for education in mathematics to serve all students well and to give them opportunities to develop their knowledge potential. Instead of teaching students to simply memorise facts and algorithms, education should be aimed at helping students to use their minds (Silver & Lane, 1993, p.14).

Material was collected through small groups, available tools (i.e the calculator) and the nature of mathematics tasks. The tasks were analysed internally and externally. Internal analysis was done by letting the students orally explain their work or by analysing the errors made in their written answers. Mathematics educators and psychologists performed the external analyses by studying the tasks and their presentation (Silver & Lane, 1993).

In Smith and Stein (1998) the findings from the QUASAR project supported the position that the nature of the tasks (exposed to the students) determines what students learn. The engagement of students at a high level is not guaranteed by just selecting and setting up high-level tasks. At first, the teacher has to consider the students' age, grade level, prior knowledge and experience and expectations for work in the classroom (Smith & Stein, 1998). Smith and Stein (1998) use four categories of cognitive demands as a second step to classify good tasks:

- Memorisation
- Procedures without connections to concepts or meaning
- Procedures with connections to concepts or meaning
- Doing mathematics

With the help of this classification, one can study the kind of thinking required by the students based on the tasks. Tasks with a lower demand imply 'memorisation' or 'procedures without connections' being used to solve them. The demands on the students in regarding thinking are different in each category. These classifications are, as Smith and Stein state, not agreed upon among all teachers. Smith and Stein present characteristics for tasks belonging to each category.

The task characteristics of memorisation are the following (Smith & Stein, 1998, p.348):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

An example of memorisation is presented below (Stein & Smith, 1998, p.269):

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?

Expected student response:
$\frac{1}{2} = 0.5 = 50\%$
$\frac{1}{4} = 0.25 = 25\%$

The response involves reproducing previously learned facts, solved without procedures and has no connection to concepts or meaning.

The characteristics of procedures without connections are presented as follows (Smith & Stein, 1998, p.348):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

The example below is, according to Smith and Stein (1998, p.269), an example of procedures without connections:

> Convert the fraction $\frac{3}{8}$ to a decimal and a percent.
>
> *Expected student response*:
>
> | Fraction | Decimal | Percent |
> |---|---|---|
> | $\frac{3}{8}$ | 0,375 | 37,5% |

The task requires some use of algorithms and no existing description on how to do the calculations exists. The importance seems to be on producing a correct answer, not on understanding what has been done. This is also visible due to the solution not needing any description.

The tasks with a higher-level demand require "procedures with connections to concepts or meanings" and the category 'doing mathematics'.

The following four blocks describe the characteristics for procedures with connections (Smith & Stein, 1998, p.348):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding

The following example is given (Stein & Smith, 1998, p.269):

Using a 10x10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.

Expected student response:

Pictorial:

x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x

Fraction	Decimal	Percent
$\frac{3}{5}$	0,6	60%

The last list of characteristics concerns 'doing mathematics', as described below (Smith & Stein, 1998, p.348):

- Require complex and non-algorithmic thinking - a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example

- Require students to explore and understand the nature of mathematical concepts, processes, or relationships

- Demand self-monitoring or self-regulation of one's own cognitive processes

- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task

- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions

- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

An example of doing mathematics follows (Stein & Smith, 1998, p.269):

Shade 6 small squares in a 4x10 rectangle. Using the rectangle, explain how to determine each of the following: *(a)* the percent of area that is shaded, *(b)* the decimal part of area that is shaded and *(c)* the fractional part of area that is shaded.

One possible student response:

Picture:

x	x								
x	x								
x									
x									

See next page...

Continued.

(a) One column will be 10 %, since there are 10 columns. So four squares is 10 %. Then 2 squares is half a column and half of 10 %, which is 5 %. So the 6 shaded blocks equal 10 % plus 5%, or 15 %.

(b) One column will be 0.10, since there are 10 columns. The second column has only 2 squares shaded, so that would be one-half of 0.10, which is 0.05. So the 6 shaded blocks equal 0.1 plus 0.05, which equals 0.15.

(c) Six shaded squares out of 40 squares is 6/40, which reduces to 3/20.

In the example, the student should explore and understand the mathematical concepts. The illustration has to be transformed into mathematical concepts via certain processes. The students need to understand the problem and the solution to explain the process.

Stein and Smith (1998) used this framework to study how tasks used in the classroom form the basis for the students' learning. The results showed that students who performed well on problem solving and reasoning tasks were those (more likely) using tasks at high levels and cognitive demands (the higher levels described before). The function of the teacher is described as a supportive factor, deciding which tasks to work with and how (Stein & Smith, 1998; Henningsen & Stein, 1997).

From the QUASAR project, Smith and Stein (1998) discuss the selection and creation of mathematical tasks done by the teacher, where it is the teacher who should select and evaluate the tasks based on his or her goals for student learning. It is also important to start with a task that (at least) has the potential to engage students at a high level to develop their thinking and reasoning abilities (Smith & Stein, 1998; Stein & Smith, 1998; Henningsen & Stein, 1997).

The presented taxonomies and frameworks are of importance when studying and discussing the conceptual demands of education, and can be used when studying teaching material such as textbooks. This framework is used as part of my developed framework for analysing differentiated tasks.

3.4 Differentiation in textbooks

Studying differentiation in mathematics education has mainly consisted of studies regarding the classroom organisation. Researchers have tried to answer questions of how one should group low and high ability students to get high achievement for all. The main focus in this work is instead the textbook and tasks as differentiating material because studies (Wallby et al., 2000; Pepin & Haggarty, 2002) have shown that content and structure of education is just as important as the organisation of students in the classrooms, if not more.

Haggarty (2002) describes differentiation as using different kinds of tasks. By giving open-ended tasks, the teacher can study the students' answers to see differences in their abilities. By giving different tasks to different students, they all work at different levels. When using pace as a measure to decide the student's ability, the faster student is often considered as high achieving. Faster students are also, to a higher degree, working with enrichment tasks to wait for the slower students.

By comparing the use of textbooks in classrooms by teachers from England, France and Germany, Haggarty and Pepin (2002) found some interesting results. When looking at differentiation, France was the only country using the same book for all students of the same age. In Germany and England, the textbooks were aimed specifically at the different ability groups in the educational structure. All three countries had different ways of organising education. In England, students were grouped by achievement (ability) into sets, such as mathematics. Education in France focused on groups of mixed ability (heterogeneous groups). In Germany, students were grouped into three different school types: Realschule, Hauptschule and Gymnasium, according to their former achievement in school.

Three levels of textbooks existed for the different groups of achievement in England. Students with high ability were said to need exercises with interesting and challenging questions, intermediate students needed straightforward questions practising skills or techniques and low ability students needed material that focused on connections to reality, a better layout and lower demands on language. In France, the teacher was responsible to select exercises from the textbook for the different levels of attainment among the students. The topic and the content of the lessons were the same, but the tasks were differentiated. German teachers (in Hauptschule) adjusted their teaching style and their use of textbooks as per the perceived ability of the students. Textbooks were used more in groups with lower abilities. Students with low abilities received short recipe-like teaching of algorithms followed by exercises. It seemed from the study that German textbooks gave low achieving students a frame and support for learning. This differed from the high achieving students who had more time regarding explanation and development of ideas.

Swedish textbooks are greatly influenced by the instructional material used and developed in the IMU project[1]. Figure 3.2 illustrates this.

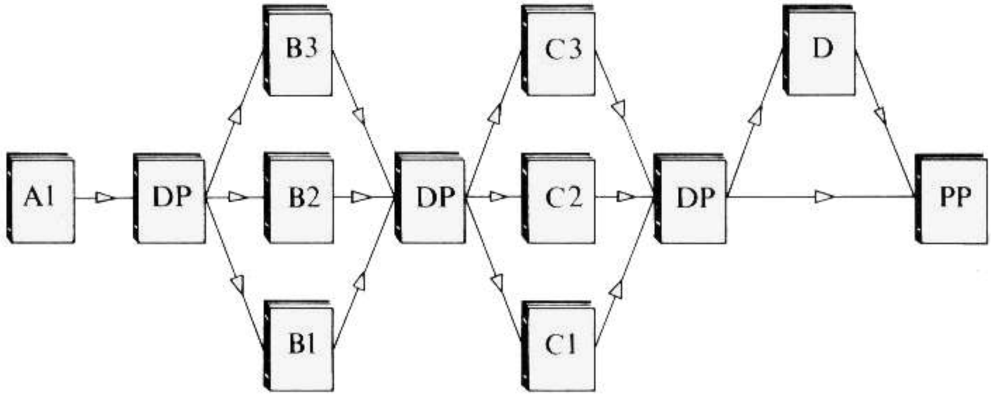


Figure 3.2: The fifth version of the IMU material (Olsson, 1973, p.7)

The figure describes one of the many developed and tested models in the IMU project. In this version, the students start with the same booklet (A1) and do a diagnostic test (DP). Depending on the result of the test, the students work with different booklets (B1-B3) and do another diagnostic test (DP). The students work through another segment like this before the final test (PP). This way, the booklets are neither organised in two alternative courses nor separated into different school years, since the students will work at their own speed. The tasks in the booklets are divided into three or four strands in the different sections, depending on which version of the material is being used. Based on the diagnostic tests, the students work with the different booklets at their own speed. During the 1960s, the material used for education in mathematics was inspired by this project (Grevholm, Nilsson, & Bratt, 1988). Resistance against a fully individualised education increased after 1969, leading to a decrease in the numbers of booklets, to finally only two books for each school year.

[1]For more information on the IMU project, see page 16

In Sweden, textbooks are often said (Wallby et al., 2000; Skolverket, 2003) to guide differentiation in the classroom, this in terms of content and format of the textbook:

> The levels, or tracks, existing in some textbooks often control the grouping of students. (Wallby et al., 2000, p.42, my translation)

If this is correct, the content of the textbooks is critical. A study is therefore needed to clarify how the tasks from textbooks are differentiated, and this is my intention with this study.

4

Methods

This study will attempt to answer both global and local questions. Globally, I have an interest in textbooks and their function regarding differentiating mathematics education. To study this global question, analysing textbook content and studying its use in the classroom is needed. This study is based on tasks and their difficulty levels.

The study is based on three textbooks for Swedish school grade 7, to be described further in this chapter. To answer the local questions (e.g. my three objectives) the material is further narrowed down to one chapter from each textbook, which happened to be the chapter on fractions because of its availability in each textbook. Learning and teaching fractions is described further in the text. The issue of differentiation is the global question, while the tasks that are clearly differentiated into different strands will answer the local questions. To study the differences in the tasks an analytic tool was constructed and is described deeper at the end of this section.

4.1 Analysed textbooks

Textbooks and their tasks are analysed in this study because of their strong connection to education and specifically mathematics education in Sweden (Skolverket, 2003; Wallby et al., 2000). I have chosen to study in more detail three textbooks used in grade 7 in Swedish classrooms. In this section, the three series and their books for school years 7 to 9, are presented, though not everything is used in the study. The analysed books are: Matematikboken (Undvall et al., 2001), Matte Direkt (S. Carlsson et al., 2001) and Tetra (L.-G. Carlsson et al., 1998).

The three books were selected primarily because their structure were partly done by placing the tasks in strands. As a background, two surveys[1] were

[1] The first is based on telephone calls to each school in one municipality and the second

conducted in 2003 and showed that Matematikboken and Matte Direkt were the most used textbooks in two Swedish municipalities. Matte Direkt and Tetra followed most of the criteria used in my review of textbooks, performed during 2002 (Brändström, 2002).

Matematikboken consists of a series of books that have been used in Swedish schools for a very long time (since 1980), longest of all the textbooks. By comparison, Matte Direkt and Tetra are relatively new textbooks. A description of these books follows.

4.1.1 Matematikboken

Authors to the series Matematikboken XYZ are Lennart Undvall, Karl-Gerhard Olofsson and Svante Forsberg. The publishing house Almqvist & Wiksell through Liber publishes the series. Matematikboken has been distributed to schools (years 7 to 9) since 1980, with the latest edition arriving between 2001 and 2002.

The series is constructed so that students with extreme difficulties in mathematics can work with a training booklet (for each grade) before continuing with the textbook for the basic course. In year 7, there exists one basic book, followed by two books for each year (the green and red book)[2] for years 8 and 9. Students decide for themselves which book they will work with, often with support from the teacher. For high ability students in need of extra material there also exists one special booklet (S). The analysed textbook is marked with a dashed square in the Figure 4.1.

There is also one problem-solving booklet (PS) in the series, containing problems of different kinds and to be used as extra material.

Tasks on fractions are represented in practically all books in the series, except for year 9, the red book. My study of the textbook series Matematikboken XYZ is limited to Matematikboken X (Undvall et al., 2001), used in school year 7.

4.1.2 Matte Direkt

The authors of Matte Direkt are Synnöve Carlsson, Karl-Bertil Hake and Birgitta Öberg. The publishing house Bonnier Utbildning publishes the book.

written communication with teachers.

[2]The green book is intended for students reaching for the grades 'Pass' (G) and 'Passed with Distinction' (VG). In the red book students intend to reach for the grades 'Passed with Distinction' (VG) and 'Pass with Special Distinction' (MVG).

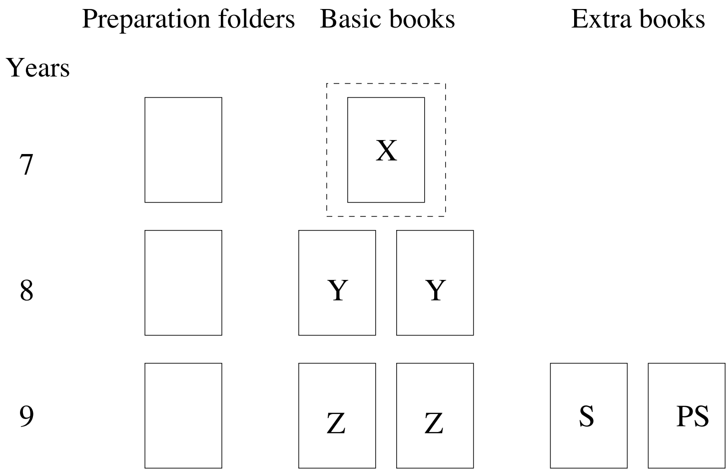


Figure 4.1: Illustration of the series Matematikboken XYZ

Matte Direkt is a relatively new textbook and has been distributed to the schools (years 7 to 9) since 2001.

The series consists of one basic textbook for each school year, as well as training books (one for each school year), extra books (S) for advanced tasks (see Figure 4.2) and a book filled with theory (no tasks) to be used as a dictionary (T).

All the basic books from Matte Direkt cover fractions. My study of the textbook series Matte Direkt is based on the book Matte Direkt year 7 (S. Carlsson et al., 2001).

4.1.3 Tetra

The authors of the series Tetra ABC are Lars-Göran Carlsson, Hans Ingves and Kerstin Öhman. The textbooks are intended for grades 7 to 9 and are published by the publishing house Gleerups. As with Matte Direkt, Tetra is in its first edition, published in 1998.

Tetra contains three training booklets, one for each year. There is one basic book for each year, and the series also contains training booklets (one for each year), an extra booklet available for students who want to prepare for continued education (S) and a DVD-book where one can listen to all the theory presented in the books[3] (see Figure 4.3).

[3]Similar material can be ordered from any publishing house if there are students with

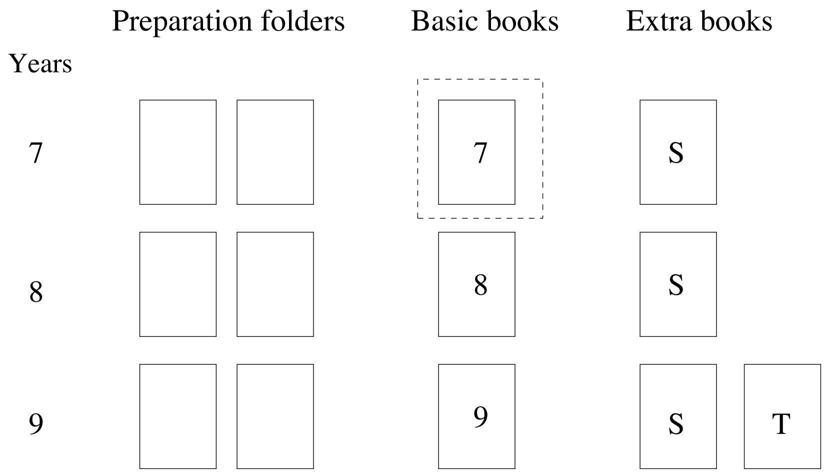


Figure 4.2: Illustration of the series Matte Direkt

All books (for all school years) include tasks on fractions. My study of the textbook series Tetra is based solely on the book Tetra A (L.-G. Carlsson et al., 1998), used in school year 7.

4.2 Fractions

To study how differentiation is handled and supported in the textbooks, this investigation focused on the mathematical subject fractions, since it was covered in all school year 7 textbooks.

Fractions take the concept of division into another dimension. The term can be described as a number or as a part of a whole. The most dominant description of a fraction is the part-whole presentation, illustrated by using chocolate bars, pieces of cake or table arrangements (Nunes & Bryant, 1996; Domoney, 2002), because the students first meet with and use fractions when, for example, sharing equally (Streefland, 1997; Meagher, 2002). However, there is more to the concept of the fraction. A fraction is also a number, describing either a ratio (x:1=3:4), a quotient (4x=3, x=3/4), a measure (e.g. one and three quarters, which is more than one and less than two) or an operator (three-fourths, stretches it three times and shrinks it by four) (Domoney, 2002; Nickson, 2000). Many researchers (Nunes & Bryant, 1996; Domoney,

hearing disorders in the class.

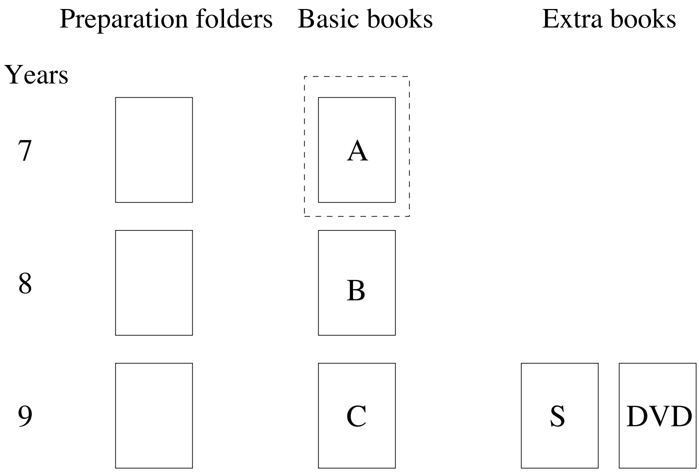


Figure 4.3: Illustration of the series Tetra ABC

2002) emphasise the importance of teaching fractions by first using the student's previous knowledge (i.e. sharing) and from that perspective, followed by its use as operators, and finally develop an understanding of rational numbers. The difficulty in teaching fractions is two-fold: to show both parts of the fraction as a number and a part-whole (Domoney, 2002; Streefland, 1997; Nunes & Bryant, 1996), while not going too fast in working with rational numbers and not understanding the concept (Meagher, 2002).

According to the Swedish curriculum Lpo94 (Utbildningsdepartementet, 1998) and syllabus, the topic of fractions is of great importance. The following are the goals the pupils should attain (National Agency for Education, 2000) at the end of the ninth school year:

> Pupils should have acquired the knowledge in mathematics needed to be able to describe and manage situations, as well as solve problems that occur regularly in the home and society, which is needed as a foundation for further education.
>
> Within this framework, pupils should have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form. (p.25)

Anderberg (1992) describes the area of fractions as useful for students who study at a higher educational level. He emphasises it as important basic knowledge, though not used very often in everyday mathematics. According to Unenge (1988) fractions were used to calculate units of measurement as, for example, scores or dozens (today we have a standard unit with decimals). He also states that a good knowledge of fractions is now needed at upper secondary school when dealing with algebraic simplifications.

4.3 Methods of analysis

I realised, after reviewing textbooks used in grade 7 (Brändström, 2002), that different structures in the textbooks when differentiating existed, which will be further described in this thesis. At the second and local level, I will look deeper into the construction of tasks arranged into different strands, requiring the construction of a framework based and inspired by different taxonomies and frameworks[4].

4.3.1 Structure of the textbooks

The structures of differentiated tasks represented in the textbooks are here presented. The chapter on fractions will be analysed and how the structure is created in the three books explained. Presenting the corner-stones of textbooks does not require any deeper theory or method; this is why its description is rather small compared to the analysis of ability levels. To get a wider view on the collected material, one can read and follow an analysis of textbooks (Brändström, 2002), where much of the information is taken from.

4.3.2 Construction of the used framework

A framework to study the differences between the tasks was constructed. A similar framework is difficult to find in the research literature. Several frameworks (e.g Blooms taxonomy) used for studying education in general and sometimes mathematics in specific were adopted, though their full usage has not always been possible but to some extent and in revised versions. How the constructed framework is used will be described here.

The analysis tool focuses on four aspects, studying the differences between the tasks (see Figure 4.4): 'use of pictures', 'required operations', 'cognitive

[4]For more information see section 3.3

processes' and the 'level of cognitive demands' present in the tasks. The aspects were independently chosen and did not originate from the same framework.

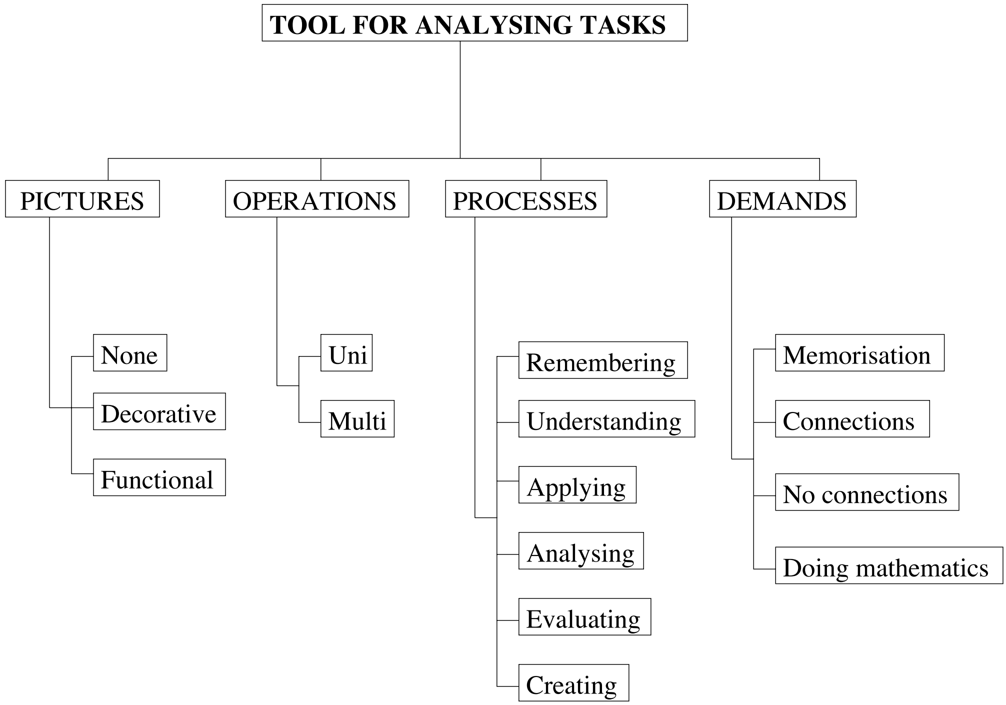


Figure 4.4: The used framework

At the beginning of this study, aspects that could be of importance when analysing differentiated tasks were listed, e.g. pictures, type of answer required and number of tasks within each strand. Some connections to the listed aspects were made in the theoretical background. With the taxonomies and frameworks as a guide the use of 'cognitive processes' and 'level of cognitive demands' were studied. Since the frameworks in these two cases are hierarchical, one might expect some relationships to tasks at different difficulty levels. There are also good reasons to believe that pictures could facilitate tasks, e.g. using more (functional) pictures in easier tasks. The number of operations is of interest because it can have a differentiating function. A task only requiring one calculation might be easier than a task requiring many. Many calculations in one task can perhaps result in many mistakes along the way, thus giving more things to keep in mind.

In the studied textbooks, only tasks aimed for students to work with have been analysed, excluding the examples presented in the introductory explanatory text or between the tasks. Only what is explicit in the tasks was studied and not what the student actually does. Many of the chosen aspects originally study the student's response to the tasks or to the expected solutions. The aspects will be used to analyse the tasks as presented in the textbooks without the involvement of any student. Other aspects that could also be of interest are, e.g. the use of text, in amount, wordings and context. Each of the following aspects will be presented theoretically.

Pictures

Pictures in a textbook can have different goals. They can either lighten up the (sometimes) difficult and theory-filled text or illustrate a problem and make it easier or more difficult to solve. Three different categories of tasks concerning the presence of pictures are defined and used in this study:

- Tasks with no picture
- Tasks with picture as decoration
- Tasks with a functional picture

'Decorative pictures' do not give any help or guidance to the tasks; they are only there as pure decoration. 'Functional pictures' illustrate the task presented in the pure text and are needed to solve the tasks. 'Functional pictures' were studied in an analysis of mathematics textbooks done in Sweden by Grevholm, Nilsson and Bratt (1988). Their historical review presented that the use of pictures was changed between 1950 and 1980, the two types (pedagogical and functional pictures) were used from 1950. In mathematics education, pictures are described as important and central (Arcavi, 2003).

Required operations

The number of operations at the different strands is also studied by organising the tasks into two groups. Tasks requiring no or one operation to solve ('uni') are in the first group, while the second group consists of tasks needing two or more operations ('multi'). 'Required operations' can also be described as the number of steps to solve the tasks. The choice of this aspect was inspired by the SOLO taxonomy (Biggs & Collis, 1982) and its categories 'unistructural' and 'multistructural'. In this study, the number of operations is analysed, while the SOLO taxonomy is partly used to study the number of closures. They are related, though not totally comparable.

Cognitive processes

The 'cognitive processes' reflected in the textbooks' tasks were analysed. This is also studied in Blooms taxonomy (Bloom, 1956) but some clear differences exist between the two methods. My area of interest is not the curriculum (for which the Blooms taxonomy is intended) or tests (as it has been used for), but instead tasks in the textbooks. Unlike Bloom's two-dimensional revised taxonomy that studies both the process and knowledge dimension, this study is one-dimensional, studying only the process dimension (Anderson & Krathwohl, 2001). This choice was made because the result of the one-dimensional study is easier to present and compare with the other aspects studied. The categories 'remembering', 'understanding', 'applying', 'analysing', 'evaluating' and 'creating' (as presented in the revised version, Table 4.1) are used in this study:

Processes	Description
Remembering	Retrieve knowledge from long-term memory
Understanding	Construct meaning from instructional messages
Applying	Use a procedure in a given situation
Analysing	Break into parts and determine how parts belong together
Evaluating	Make judgements based on criteria and standards
Creating	Put (or reorganise) pieces together to a whole

Table 4.1: Cognitive processes in the framework

Order and names of the categories from the revised taxonomy are used, but the descriptions are from the original taxonomy. Because I agree with the discussion on the hierarchical order, 'creating' should come after 'evaluating'. Also, the names seem to be more appropriate to the intentions of the study, since they should reflect the intended thinking and acting (of the student). An action is easier to reflect upon as a verb, as also described in the revised version of Bloom's taxonomy (see section 3.3.2).

Since the same task can belong to several of these categories, they have been labelled according to the highest process required in the specific task. Because it is based on a hierarchical view, this is how the taxonomy is used in both the original and revised versions.

Level of cognitive demand

To study the thought process of a student when solving a task, the aspect 'level of cognitive demands' is used. The framework described by Smith and Stein (1998, p.348) is used in full. In their framework, four aspects are used to analyse the level of cognitive demands of the tasks:

- Memorisation
- Procedures with connections to concepts or meaning
- Procedures without connections to concepts or meaning
- Doing mathematics

The original framework did not require any modifications to fit my study, since its use had been thoroughly described by the authors, providing both theoretical information and examples of analysing mathematical tasks.

'Memorisation' is demanded when the student does not have to use an algorithm to give the correct answer. A 'procedure without connections' requires algorithms, but does not give any meaning or relate to any concept to what is being learnt. 'Procedures with connections' give a deeper meaning to the tasks, and often use pictures and illustrations to help the student understand. 'Doing mathematics' is the highest level of demand. In this aspect, the tasks often require more than just using a method and finding the solution. Different concepts are often required for the students to put together the answers by themselves and solve the task.

4.3.3 Using the framework

To study if the constructed framework could be used, a smaller analysis with one of the textbooks lead to some minor adjustments to the aspects. In the aspect 'number of operations', I realised that there was no need to use too many categories. Therefore, only two ('uni' and 'multi') were used. Difficulties were encountered when analysing the 'cognitive processes' needed in the tasks because of the lack of a hierarchical view from the beginning.

How the constructed framework was actually used is presented in the following text. As a starting point, the tasks based on each aspect are studied and compared with those in each strand.

A presentation of three examples with tasks from the textbook Matematikboken X (Undvall et al., 2001) follows. Although these facts are not presented in the text, the tasks used in the examples are from all different strands and are therefore of varying difficulty. The three examples below focus on each task and cover all the categories of the framework to show how the tasks in each aspect are analysed and how the framework can be used to analyse the tasks using all the aspects together.

Example 1:

5140. How many seconds is a) 1 min (p.238, my translation)

My comments:

- **No picture** is connected to this task. A student can use a clock to illustrate it by him- or herself though this is not done in the book. Only what is given or required in the task regarding pictures is presented. If there had been a demand to draw a picture the task would have been analysed as having a functional picture.

- The task requires no algorithmic operation since the student uses his or her memory to give the right answer, i.e. it is a **uni** task.

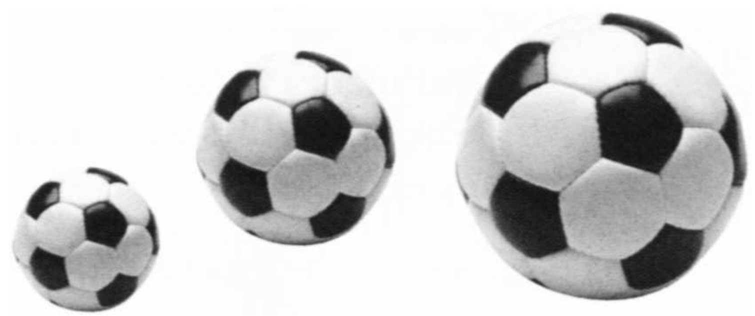
- The process required to solve the task is **remembering**. The next step in the hierarchy is the process of understanding what is being asked for to use the right method for solving the task. This is not required in this case.

- The level of cognitive demand is **memorise**. No procedures can be used in this task other than present what has been memorised from beforehand; in this case, that one minute is 60 seconds.

Many of the aspects used in this example are closely connected to each other. The 'cognitive process' and 'level of cognitive demand' are in this case the same, since the student is only required to remember a formerly known fact.

Example 2:

5068. One year, Moas football club received 18 000 SEK in contribution from their municipality. 7500 SEK was used for the purchase of football clothes and balls and 4500 SEK for travelling expenses. How big part of the money was used for other expenses? Answer in the simplest form. (p.221, my translation)



My comments:

- The picture of footballs is purely **decorative**. They are not connected to the task in any mathematical way and are therefore not to be used in order to solve the task.

- The task requires more than one operation, making it **multi**-operational. To solve the task, the student has to calculate how much money remains after all expenses and present the result in the form of a fraction in its simplest form.

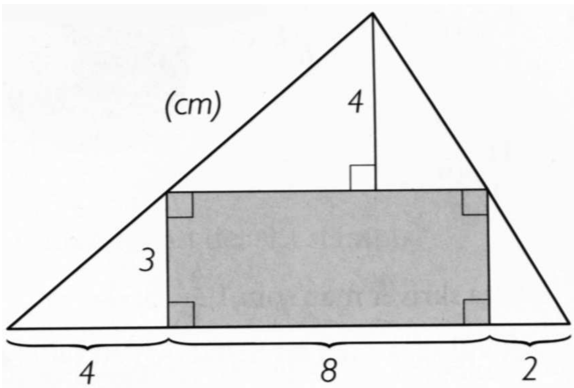
- The processes required to solve the task are 'remembering', 'understanding' and 'applying'. The "highest" process used is **applying** and is therefore the label of the task. The situation is presented; the student should subtract the expenses from the given sum and then present the result as a fraction in its simplest form. This involves using many procedures to given situations defined as applying.

- This task is classified as using **procedures with connections to concepts or meaning** because of the effort needed to solve the task and that it is based on a problem situation. Pathways to solving the task are not explicitly given and the use of procedures is in focus. The task does not involve any functional pictures or symbols, but does require some cognitive effort since the instructions cannot be followed mindlessly.

The aspects in the framework show that this task has some difficulties. However, it becomes easier once all numbers are given and the implicit pathways are clarified (e.g. the methods are found).

Example 3:

5026. How big part of the triangle is coloured? (p.213, my translation)



My comments:

- The task contains a picture that is needed to answer the question, i.e. a **functional picture**.

- The number of operations required is more than one, making it a **multi-**operational task. The main task is to compare the two areas. Before being able to compare them, the sides in the figures have to be calculated by using some of the given numbers followed by the two areas being calculated.

- Numerous processes, if not all, are required to solve the task. The student needs to break up the task into subtasks and then connect them to each other to solve the actual task, by calculating the sides that are needed, the areas and finally how much that is coloured. The student needs to know how to calculate not only fractions, but also the areas of a triangle and a rectangle. The highest process needed is therefore **creating**.

- The level of demands in this task is **doing mathematics**. The student should explore the picture and understand the mathematical concepts needed by transforming the picture into calculations. There is no suggested approach solving the task; the student needs to explore the picture and find the constraints.

According to the framework used to analyse the task, slightly more is required from the student. Many concepts and meanings are not given in the text (e.g. terms like area, triangle and rectangle), though the student makes the connections between them. It would be interesting to see how students of different ability levels, with the teacher giving some guidance for students with problems, actually solve the task and what kind of solution they present.

Comments on the use

The constructed framework provides information on separate aspects, such as pictures, operations and demands as well as the total presentation of the task. In the basic part of the analysis, the separate aspects question the construction of each task, e.g. by presenting the 'use of pictures' or 'level of cognitive demands'. The second function of this framework (and maybe the most important) is that it can be used to discuss the difficulty level of the task when examining all aspects together.

The categories at the bottom-level of the two aspects 'cognitive processes' and 'level of cognitive demands' are very similar. Both aspects focus on what the student should do to solve the task, though the aspect 'level of cognitive demands' has an additional requirement. It also includes a study of how the task solution is presented. The top-level categories are rather different since the additional demand has a significant role in the tasks.

4.3.4 The methods and their limitations

This chapter has described the selections and choices for the design of the method.

The general question of how textbooks differentiate is rather broad, and has lead to a study on how differentiated tasks vary in levels of difficulty by using several aspects. It has also lead to questions on the differences between strands, with respect to what aspects are used and to what degree. Other questions have concerned the grouping of students: why and how it is done, and the students' time constraints for working on the tasks. The latter questions could not be answered within this study.

As described earlier in the chapter, the chosen aspects are limited. However, this number could increase and a study on the use of text or mathematical context in the tasks might follow this study, yielding additional results.

As earlier described, the choice of textbooks is mainly based on their use of differentiated tasks. Other textbooks or chapters could also be analysed to increase the amount of tasks analysed and to get an idea of a larger group of textbooks. One could, for example, ask if the chosen chapters are similar to other chapters in the textbooks. The results and conclusions of this study would then apply to them as well. Teachers and authors of the textbooks could then consider this study when using and constructing a textbook.

The difficulty of this work has been to study tasks without using the students' solutions. The analysis of the tasks is based upon how they are written in the textbooks (by the authors) and my interpretation of the tasks and their solutions. My intention is to follow up the analysis with interviews of teachers and students and observations in the classrooms, to see if my conclusions are valid when the textbooks are used.

4.3.5 Reliability

The term reliability is used when one wants to know if a method can be repeated with similar results. One way to test the reliability of the tool (or the analyser) is to analyse the tasks on more than one occasion by the same analyser and then compare the results. Intercoder reliability is a common way to analyse the reliability in content analysis (Kaid L. & Johnston W., 1989), providing an indication of the stable and enduring characteristics of the analyser. To compare the two test results and assess intercoder reliability, Holsti's formula for computing reliability was used. The formula uses coefficients based on a ratio of agreement among analysers. In this study, a strand in Matematikboken X (basic) consisting of 192 tasks was reanalysed. The study was performed with the same analyser and the same procedure as the original study, but with a time difference of five months.

Holsti's formula for computing reliability is as follows:

$$R = \frac{C_{1,2}}{C_1 + C_2}$$

$C_{1,2}$ = Number of category assignments both coders agree on.
$C_1 + C_2$ = Total category assignments made by both coders

For each category, the numbers of tasks analysed yielding the same results are calculated. These are divided with the total number of tasks analysed. According to Lee Kaid and Johnston Wadsworth, researchers can usually be satisfied with coefficients over + 0.85, and react to coefficients below + 0.8. For the data in this study, the reliability coefficients are:

Aspects	**Reliability**
Use of pictures	+ 0.95
Required operations	+ 0.96
Cognitive processes	+ 0.91
Levels of cognitive demands	+ 0.93

Table 4.2: Reliability coefficients

The Holsti's formula shows high reliability coefficients between the two tests. All coefficients are above 0.9; therefore the reliability of the analyser is high in this specific study. For more information on the data, see Appendix D.

5

Results

This chapter presents the results of the analysis. A description of how the chapters on fraction are structured in the three textbooks is presented, followed by presentations on the differences between the ability levels in the four aspects: ‘pictures’, ‘number of operations’, ‘cognitive processes’ and ‘levels of cognitive demands’. In all sections, interesting results from the study will be described, both based on individual textbooks and by comparison. The results and conclusions of using the constructed tool are presented at the end of this section. The results of the study are summarised at the very end of the chapter.

5.1 Structure of differentiation

Textbooks can use different methods to differentiate. The structures used in three Swedish textbooks follow in this section. The illustrations are to be read from left to right and the blocks are not representative of their relations to each other in size. There is also a comparison between the structures at the end of this section.

5.1.1 Matematikboken

The studied chapter in this book is constructed with the help of several sections of tasks[1] (Fig 5.1). It starts with a presentation of the basic knowledge to learn and tasks to solve, where the tasks are grouped in three strands (A, B and C) based on their difficulty, as presented in the following section. A section of applied tasks (problem solving) follows, not ordered in specific strands, but with different sets of goals. For example, activities to be done in smaller groups to develop the student’s communication skills in mathematics might exist.

[1] For the whole series Matematikboken see section 4.1.1

After these two sections, the textbook hints at a diagnostic test for the students to do and test their knowledge of the subject to date. This is not found in the textbook, but in the teacher's tutorial for this specific textbook. Based on the diagnostic test, the students' continued work could be decided. The following tasks are grouped in two strands. If the student fails the diagnostic test, the continued work will then be based upon repetition tasks (one of the strands); and if the test is passed, there are continuing tasks considered more demanding to work with (second strand).

<table>
<tr><td>Basic course A–level</td><td rowspan="3">Mixed tasks</td><td rowspan="3">Summary</td><td rowspan="3">Diagnostic test</td><td>Repetition tasks</td><td rowspan="3">Mixed tasks</td><td rowspan="3">Test</td></tr>
<tr><td>Basic course B–level</td><td rowspan="2">Continuing tasks</td></tr>
<tr><td>Basic course C–level</td></tr>
</table>

Figure 5.1: A chapter in Matematikboken

After the follow-up section, as I have called it, follows another section of applied tasks/problems solving tasks. These tasks are not grouped into strands based on their difficulty. Some tasks for repetition followed by the final test are at the end of the textbook chapter. The final test can be found in the teacher's tutorial.

5.1.2 Matte Direkt

Every chapter in Matte Direkt starts with applied tasks as an introduction to the subject[2] (Fig 5.2). After these tasks, the basic course starts. The basic course (the green course) is not divided into different strands, and all students work with the same tasks. To test the students knowledge on the basic course, the textbook uses a diagnostic test from the teacher's tutorial. After the diagnostic test the tasks are divided into two strands (the blue and red course). The tasks in the blue course are for students who did not pass the diagnostic test, while the red course is for those who did.
A summary in written text is followed by a section of mixed tasks (applied tasks). The knowledge gained by the student is tested in the final test that is also found in the teacher's tutorial.

[2] For the whole series Matte Direkt see section 4.1.2

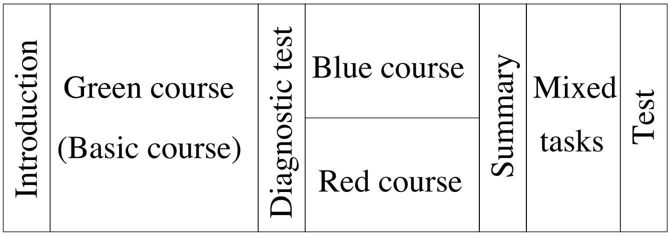


Figure 5.2: A chapter in Matte Direkt

5.1.3 Tetra

The chapter on fractions in Tetra[3] starts with an introduction consisting of some applied tasks (see Figure 5.3), followed by a basic course with theory and tasks.

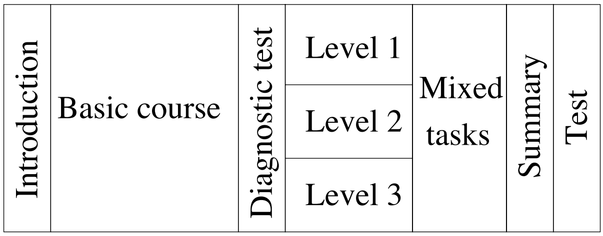


Figure 5.3: A chapter in Tetra

This section does not use strands to group the tasks and the students work with all of them. A guide to the diagnostic tests that can be found in the teacher's tutorial follows, and a section with grouped tasks are next (strands 1, 2 and 3). Students who do not pass the test work with tasks in the first strand, which are repetitions on the theory given before the diagnostic test. When the students are finished with the first strand, they proceed to a second diagnostic test. Those students passing the first or second diagnostic test continue with tasks on the second strand followed by the third strand (with a described increasing degree of difficulty). Some activities to be done in groups come after, then a summary (in text) to end the chapter before the students take the final test. The final test is to be found in the teacher's tutorial, as the diagnostic test.

[3]For the whole series Tetra see section 4.1.3.

5.1.4 Summary on the structure

The three analysed chapters have similar structures. In general, they all start with a basic section, followed by a diagnostic test, to test the knowledge attained by the students, and a follow-up section.

The basic sections are not differentiated into different strands in two of the textbooks, with the tasks being divided into three strands in the third textbook. The strand the student works with in the follow-up section is based on the result of the diagnostic test. These sections are divided into two or three levels in all the books. In one of the textbooks the students can take another diagnostic test (after the first strand) to ensure they have completed the basic course. At the end of one or two chapters, there is a final test to verify if the students have passed the course. Figure 5.4, illustrates some of these similarities.

Figure 5.4: A chapter

Setting different tasks to different students is differentiation. Some books also have a basic course with the same tasks to all students. Some tasks can be seen as enrichment tasks for the students to work with if they have time before the diagnostic test or the final test. This solution avoids students having to wait for each other. These tasks were not studied further in the analysis.

5.2 Tasks in strands and textbooks

The differences between tasks in the separate strands (ability levels) and the different textbooks are presented in this section. The numbers of tasks studied are not the same for each level. They are distributed according to Table 5.1. Tasks marked as 404 a, b and c, are analysed as individual tasks, i.e. a total of three tasks. As one can see and as previously mentioned, some differences exist between the textbooks and their strands. The results are presented based

	A	B	C
Matematikboken X (basic)	120	192	77

	Training	Deepening
Matematikboken X (follow-up)	76	10

	Blue	Red
Matte Direkt 7 (follow-up)	112	88

	1	2	3
Tetra A (follow-up)	143	41	74

Table 5.1: Number of tasks

on the aspects analysed. All the diagrams from the analysis can be found in Appendix C.

5.2.1 Use of pictures

In this aspect, three categories are used: no picture at all, decorative pictures not connected to the tasks mathematically and functional pictures mathematically connected to the tasks.

Clear differences between the strands with high and low difficulty are visible in the textbook Matte Direkt 7 (see Fig. 5.5).

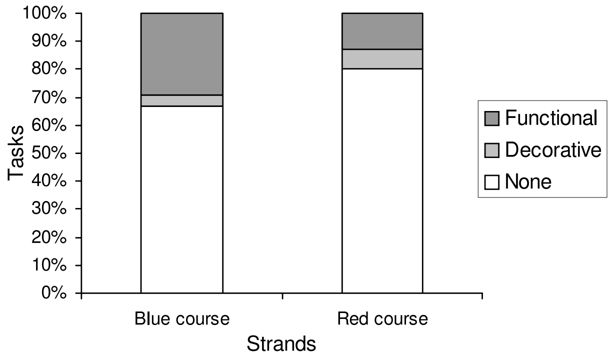


Figure 5.5: Pictures (Matte Direkt 7)

The strand with low difficulty has more 'functional pictures' than the higher difficulty. This tendency exists in Tetra 7, but just not as sharp (see Fig. C.4 in appendix C). When comparing the textbooks, the results show Matematikboken X (basic course) to have an equal amount of 'functional pictures', but to a lower degree in total (see Fig. C.1).

The category mostly shown in the analysis is that with a non-existing picture. More than half of the tasks (in all strands and textbooks) have no picture connected to them at all, neither 'decorative' nor 'functional'.

5.2.2 Numbers of required operations

Here, the analysed aspect is the number of operations used in the tasks. The two categories used are 'uni' and 'multi'. 'Uni' is for the tasks with no or one operation needed and 'multi' for those with more than one operation needed.

In almost all strands of the textbooks there is an increase in operations when the difficulty is said to rise, both in the basic section of Matematikboken X and the follow-up sections of the other books (see Fig. 5.6).

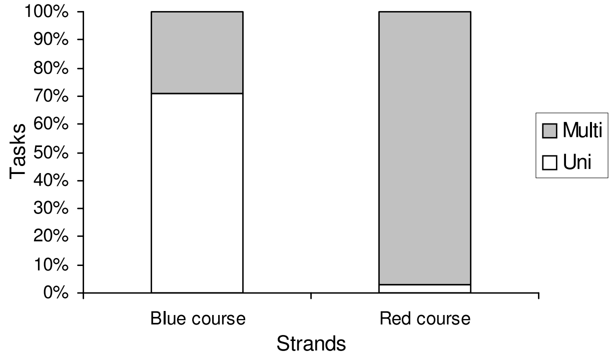


Figure 5.6: Operations in Matte Direkt 7

The figure clearly illustrates that the first strand mainly consist of tasks with one or no operation, while the second strand (opposite to the one before) requires more operations.

An exception to this can be found in Matematikboken X (basic course) and the B-strand (Fig. C.5), where the B-strand has a smaller amount of 'multi'-tasks than the two other strands, though the strands are explicitly described as having increased difficulty. The textbooks do not show any large differences between each other in this aspect.

The general result from the analysis shows the tasks belonging to lower strands requiring fewer operations than the other strands (said to be more difficult).

5.2.3 Cognitive processes

When studying the cognitive processes found in the tasks, parts of Bloom's original and revised taxonomy were used as a framework. The categories are 'remembering', 'understanding', 'applying', 'analysing', 'evaluating' and 'creating'. They are hierarchically used since the first category is often used in the ones that follow. This aspect is used to analyse what processes are required for the tasks to be solves[4].

For all textbooks, the categories at higher levels in the hierarchy are used more in the strands with (as presented) a higher difficulty level. This is clearly shown below in Figure 5.7 for Matematikboken X (basic).

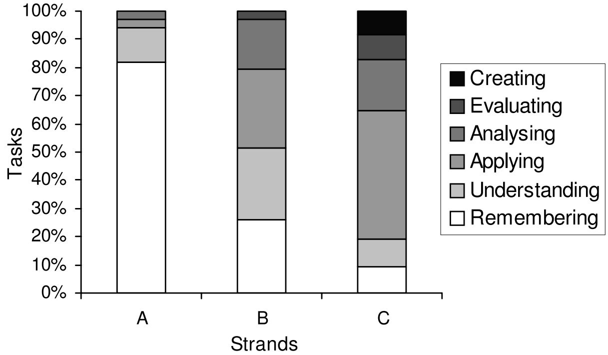


Figure 5.7: Cognitive processes (Matematikboken X, basic course)

When examining the differences between the textbooks, one result shows Matematikboken X (follow up) (Fig. C.10) and Tetra A (Fig. C.12) to use more categories on lower strands than the rest of the textbooks, i.e. they have other structures of differentiation than other textbooks.

In this picture, the strand on lower level has more categories higher up in the hierarchy than other textbooks and their strands. Higher difficulty in the strand implies less use of categories, according to the analysis of this textbook.

[4]For more information see section 4.3.2

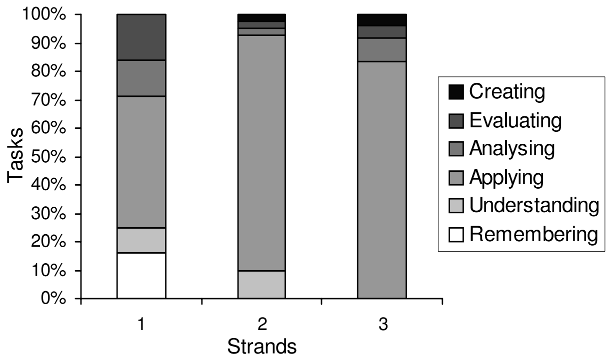


Figure 5.8: Cognitive processes (Tetra A)

The textbooks mainly use higher processes in the strands considered to be of higher difficulty.

5.2.4 Level of cognitive demands

Four categories are studied in this aspect: 'memorisation', 'no connection', 'connection' and 'doing mathematics'. 'Memorisation' and 'no connection' are lower demands than the other two. Tasks with low demands often require short answers with very little activity for the students. The tasks using higher categories require more from the students, mostly when it comes to describing what they did to find the answer.

The results of this analysis show that Matematikboken X (follow-up) (Fig. C.14) and Matte Direkt 7 use similar ways to organise the tasks (Fig. 5.9)

Both strands have mainly tasks with lower demands; the difference between the strands (in both books) is that there are more tasks with 'no connections' in the strand with a higher difficulty level. When comparing Matematikboken X, basic and Tetra 7 (Figures 5.10 and 5.11), the strands at the highest difficulty level are almost identical in their usage of categories. The main category used in both textbooks is 'no connection'. The lower strands show more differences than the higher.

In Matematikboken X (basic section) the lowest strand mainly consists of tasks with the category 'memorisation'. In Tetra A, a variety of demands are found in the lowest strand, with more usage of tasks with 'no connections' as demand. The hierarchical levels are not used as in Matematikboken X (basic

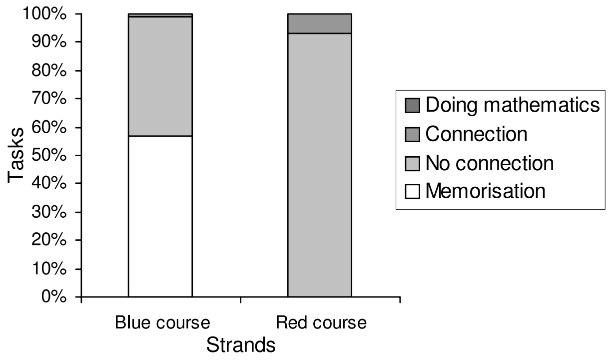


Figure 5.9: Level of cognitive demands (Matte Direkt 7)

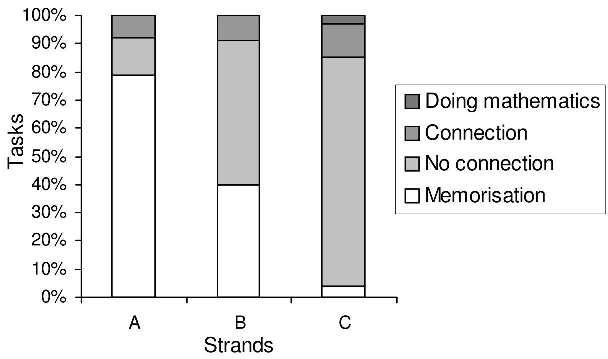


Figure 5.10: Level of cognitive demands (Matematikboken X, basic course)

section).

5.2.5 Summary of the analysis

Some clear results emerge when studying the levels of difficulty in the strands. Regarding the 'use of pictures', 'functional pictures' in two out of three textbooks are used more frequently in the strands with a lower level of difficulty than in those with a higher level.

The next aspect, 'number of operations', can clearly be connected to the difficulty level of the strand, since the number of operations increases with the difficulty level of the strand in all analysed sections of the textbooks.

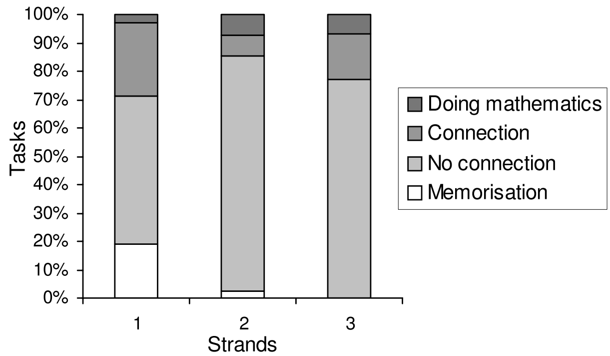


Figure 5.11: Level of cognitive demands (Tetra A)

Both the aspects 'cognitive processes' and 'level of cognitive demands' can be regarded as measuring the level of cognitive difficulty in the tasks, and the results show the strands to be mainly arranged according to this. Categories with low difficulties, such as 'remembering' and 'memorise' are represented much higher in the lower strands than in the other strands, though the "higher" categories are not greatly represented. Mean valued categories (such as 'applying' and 'no connection') seem to have a majority in those tasks. There are a very low percentage of tasks with a high level of cognitive difficulty.

When comparing the strands and their variation of the categories in the two last aspects there is mainly one category in the majority. For lower strands, these are 'remembering' and 'memorise', while the higher strands consist of tasks with the process 'applying' or the demand 'no connection'. The higher strands have more variation in the categories. One textbook shows contradictory results, due to it having a larger variety of tasks in the lowest strand.

Common to all textbooks in the study is the fact that the less demanding categories in the analysing tool are used more than the higher ones.

5.3 The tool

Constructing a tool to analyse tasks in mathematics textbooks was a goal of this work. The tool was needed to study the tasks and find differences between the strands. All four aspects used in the tool show results that can be discussed further, concerning the use of these textbooks in the classroom as well as in the development of new textbooks and material for mathematics education. Since none of the authors were contacted it is unknown if they use similar tools in constructing their tasks and organising them into strands.

The structure of the analysing tool is presented in Figure 4.4 and how it is used is described in the methods chapter (ch. 4). An analysis of the first two aspects (pictures and operations) went rather quickly compared to the last two. The aspect 'cognitive processes' was rather difficult to use. The analysis of tasks had to be conducted in several shifts to see if the categories could be used in Swedish textbooks and then compare the tasks by answering questions on why a task was higher in the hierarchy than other tasks, and finally if the results could be reasonable. The fourth aspect was based on an existing framework used in the QUASAR project, whose characteristics were clear (with descriptions and mathematical examples) and therefore easier to use.

The four aspects were analysed separated from each other, rendering very interesting results of the cognitive categories, since they were very similar when compared to the textbooks. Figures 5.8 and 5.11 show this similarity.

The tool proved to a certain degree that differentiation occurs in the studied aspects. It has (in its present condition) only been used for analysing tasks and not for the construction of tasks. The second type of use (construction of tasks) could give it an additional dimension, but in that case should include more aspects such as the use of text and mathematical concepts.

6

Discussion

The purpose of this study is to analyse tasks in mathematics textbooks and find out how they are differentiated in difficulty. The content of the textbooks is of interest since they can form mathematics education. As described earlier, this is an introduction to a more comprehensive study on mathematics textbooks. The results can therefore lead to questions to be answered later on.

6.1 Quality in research

Lester and Lambdin (1998) describe worthwhileness as the most important criterion for a good quality in research within mathematics education. To accomplish the criterion of worthwhileness the study should (p.12):

- Generate good research questions
- Contribute to the development of rich theories of mathematics teaching and learning
- Be clearly situated in the existing body of research on the question under investigation
- Inform or improve mathematics education practise

Furthermore, they offer five useful criteria to evaluate the quality of mathematics education research. The questions raised and methods used in the study have to match (coherence). A study should be relevant, carefully designed and reported and be competently carried out (competence). Personal biases and assumptions should be made public, as well as information on the collection, use and analysis of the data (openness). When using information collected from individuals (e.g. students and teachers), issues on confidentiality and accuracy are fundamental, and credit should be given to contributors of the project or study in different ways (ethics). Finally, findings should be grounded in the data and conclusions should be justified (credibility).

By using the quality criteria, as presented by Lester and Lambdin (1998), the quality of the work can be evaluated. Throughout this work the issues of worthwhileness, coherence, competence, openness, ethics and credibility have been taken into consideration.

The importance and worthwhileness of the study have been in focus from the beginning. A study like this has implications on education and the creation of textbooks. Questions on how tasks are differentiated have, for example, not been discussed as often as the organisation of students in different groups. Since a formal review of textbooks does not exist, the results are of interest for teachers as well as textbooks authors. A deeper discussion on the implications of this work can be found in section 6.3.

When constructing an analysis tool for tasks, it is important to discuss if the work measures what is intended, i.e. the validity of the tool. In this case, the study is coherent. To assure the validity of the study, the tool and results were presented in a seminar consisting of doctoral students and researchers in mathematics education. I received questions and comments that assisted in the continued work. A closer review of the tool would be a natural continuation to validate it further.

Regarding the issue of competence, this study is constructed, reported and performed in the best possible way. The construction and use of the tool are presented with connections to the theoretical background and tasks from the analysed books.

Transparency in the study makes it possible for other people to do a similar study. In this case, the analysed tool is structured and presented in the method chapter, with descriptions and examples. Adding to this, two aspects used in the tool are based on frameworks from other tools, where most of the credit is to those who have constructed them (as described in the method chapter). By combining them with other aspects a tool to study several aspects of differentiated tasks has been constructed and described as detailed as possible.

Good credibility is also needed to know that the analysis is done correctly and with trustworthy results. A trained analyser is optimal in a work like this. Two analysers conducting the same study, instead of one is also good. When calculating the intercoder reliability, the correlation was over 0.9, indicating that the analyser did almost the same analysis the second time, with some time difference (for more information see section 4.3.5). A test can be done, to find out if a similar tool can replace it and receive a similar result. Since three out of four aspects are intended to analyse students' response when solving a task, a study of the students' solutions could be one way of verifying the conclusions of this study, though this kind of study is not within the scope of this study. The findings and conclusions in this work are solely grounded in the collected data. Questions not connected to the actual results have been partially described with suggestions for further research (see section 6.4).

6.2 Results

The study has yielded many results concerning structure and content of the textbooks, as well as the constructed tool used for analysis. Here, the results are discussed in the same order as the objectives, with some connections to the theoretical background on differentiation and textbooks.

6.2.1 Structure of textbooks

In all three textbooks, the tasks at low-levelled strands are described as repetitions of the basic course, or the lowest knowledge needed to pass the test. The strands are described as more challenging the higher the strand. Similar results were presented in a textbook analysis from England (Haggarty & Pepin, 2002). These textbooks did not contain strands (as with the Swedish), but instead were comprised of three textbooks, one for each difficulty level. In the Swedish textbooks, the strands totalled two or three, with different working schemes for the students. This can influence classroom organisation if the content is different in the strands or if the students work faster or slower. According to the presented structure of strands in the books, students with different abilities (or difficulties) should be challenged at "their level". Connections can be made to the grouping of students in the classroom and in the textbooks. There can be difficulties in grouping the students as presented from the results of surveys done in England during 1970 and 1980 (Hart, 1996). Although students were grouped by ability, the teachers aimed the education at those students in the middle or below, resulting in reduced opportunities and less challenges for big parts of the group. Connections between that study and this one are difficult to make at this time of the study. As a continuation, it might be of interest to see what is happening in Swedish classrooms in that area.

The material produced in the IMU-project (Olsson, 1973) used a model similar to the analysed books, to make individualisation possible throughout the textbook. The students were then supposed to work with custom-made tasks at their own specific levels and individual speeds. The problem raised in the project concerned the actual teaching, since teachers had a less important function and it was up to the content of the textbooks to maintain the student's educational base. The difficulty level of the tasks was never raised as an issue. Swedish evaluations of education in mathematics illustrate interesting results (Skolverket, 2003, 2004): The most common form of education in the mathematics classroom is individual work with the textbooks as the prevalent material. Since this study is only based on the structure and difficulty level of the differentiated tasks in the textbooks, a connection to their use in the classroom cannot be made at this time. Questions can be raised on how and if the students' specific needs are fulfilled and what educational base they receive. This is the object of a different study.

6.2.2 Differences between strands

When studying the differences between the tasks in different strands the results can be discussed with several foci.

The variations of categories in each aspect from the textbooks and their strands are very similar. For example, the 'use of pictures' is not presented as a clear indicator of differentiation. This can be connected to the content of the chapter (fractions), but it can also be as presented without being used for differentiating tasks.

As the second aspect shows, the 'number of operations' is presented as clearly differentiating the tasks because of the differences between the low and high difficulty strands.

In the last two aspects ('cognitive processes' and 'level of cognitive demands'), there were more demanding tasks in the strands said to be more difficult. Students who work in the different strands therefore meet different processes and demands. The problems arise when looking into the lower strands, which have a majority of tasks with processes and demands requiring very little from them, mainly 'memorise' and 'remembering'. The study does not indicate if this is special for this specific chapter. As described in the chapter on teaching fractions (see section 4.2), education can start from the students reality, using their experiences as an introduction. This might be one explanation of this result.

One exception can be found among the books. Tetra 7 varies its categories in these aspects at the lowest strand more than in the highest strands. An interesting question would be the actual use of this textbook compared to the others - does this structure of tasks require more from the student and the teacher? Another question concerns the clear difference between the first and second strand in this specific textbook and again the use in the classroom.

As previously discussed, the level of challenge in the tasks at the low strands is very low. The result also presents a low amount of tasks with more challenge in the top strands. This is noteworthy since some of the categories indicating more challenge are 'doing mathematics' and the process 'create'. When summarising the goals of mathematics education, challenging tasks are very important. As described in the framework used by PISA (see section 2.1.2), mathematical competence can be measured by mathematical calculations, connections for problem solving, as well as generalisation and insight to mathematics. For textbook tasks, the student should be able to find the solution, calculate the answer, present a structured calculation and reason for the answer to show that he or she has understood, all this with the intention of using what they have learnt in other surroundings. This would imply a high variety of the analysed categories in all strands, which is not the case in this study.

The dimensions that are difficult to say anything about are the mathematical contexts and importance of text, because the analysing tool did not consider them. One must not forget that they can have important implications. Several studies have discussed the effects created by the text, e.g. Möllehed (2001) and Österholm (2004).

6.2.3 The analysing tool

Differentiated tasks in textbooks are analysed with the help of a tool based on four aspects: 'use of pictures', 'number of required operations', 'cognitive processes' used and the 'level of cognitive demands' in the tasks. Their importance concerning differentiation can be discussed.

The 'use of pictures' was less important than expected from the beginning. It was surprisingly low in all tasks, regardless of which strands the tasks belonged to. According to Arcavi (2003), visualisations (pictures) are widely accepted as important and central when learning and doing mathematics. They can be seen as key components in reasoning, problem solving and proving. This can be a result of the selection of content to analyse. There should be less illustrative pictures when calculating with fractions compared to, for example, geometrical tasks.

The number of 'required operations' was identified as important when concerned with difficulty of the tasks and strands. All results presented a low number of operations in the strands at a low difficulty level, and a high number in those at a high difficulty level.

The last two aspects, 'cognitive processes' and 'level of cognitive demands' can be analysed separately, but could also yield some important results to the study when combined. When separated, the analysis of the processes can illustrate which 'cognitive processes' are needed to solve the tasks, and what the students need to do to solve the task. The level of 'cognitive demands' shows what demands are required to solve the task, i.e. connected to the used methods and the student's answer. Together, these aspects can present an image of a challenging task level for a student. Adding the second aspect, 'operations required', will give even more information on the level of challenge. The 'use of pictures' was not important when differentiating tasks.

6.2.4 Further discussions

Earlier in this work, terms such as mathematical literacy, competence and proficiency were described and together with the Swedish curriculum, all point into the same direction. Learning mathematics should be done in an active process, by preparing students for the challenges they will meet in the future. Students should be able to reason, communicate and reflect with the help of mathematics, as well as be able to use and understand a mathematical language, ask and give answers to problems, make historical connections and be able to use technical tools. Mathematical confidence is most important according to the syllabus. When studying the results with these comments in mind, the last two aspects could be expanded. If the comments above were applied to textbooks and their tasks, the tasks should be more of the categories 'creating' (in cognitive processes) and 'doing mathematics' (in 'level of cognitive demands'). For education to be equivalent to the curriculum, the teachers need to bear this in mind when planning their lessons. This study is not an analysis of the entire book, making it impossible to draw any conclusions on that. Teachers should be aware of this result when it comes to the analysed section.

6.3 Implications for teaching

What the students actually learn and how we can develop teaching to help all students achieve better is of utmost importance. The grouping of students by ability or mixed ability is, as mentioned in the text, a political issue often described as two-sided. Either this is described as education for all, with a focus on a highly-educated mass of citizens, or the focus is on the demands of a small group of top educated citizens. All students should be considered in terms of education for all and differentiation. Therefore, this study claims that students with low and high abilities should have the opportunity to advance in their mathematical knowledge; education should not only focus on one group.

According to this study of the different strands, the tasks are not adjusted to either side (low or high abilities). The textbooks are said to consist of differentiated tasks, but according to this study, this is not done as intended in the curriculum or the research on differentiated learning. At all levels, the processes and required demands are too low. The implications that the result of this study have on student's achievement are difficult to predict, since the content of the textbook and its use can vary greatly.

Another important result of this study is the construction of the analysing tool. By connecting the 'required operations', 'cognitive processes' used and the 'level of cognitive demands' to solve the task, a concrete and useful tool has been developed.

The three textbooks analysed in this study are different in some points, and similar in others. If the tasks in the textbooks are not challenging enough for the students, it is up to the teachers to include material and problems for the students. I believe that textbooks are very useful, both for teachers and students, but are not containing all information to do good teaching and learning.

Textbooks with content presenting the nature of mathematics and encouraging students with challenging tasks can result in other problems. Pedersen (1995) analysed the use of two Danish textbooks to study the demands from the school setting on the teaching material when concerned with the nature of learning mathematics. The two textbooks were chosen because they contained content discussing the importance and meaning of mathematics and encouraged students to learn by doing. When using the two textbooks, Pedersen realised that her image of a good textbook was very difficult to use. The poor book (according to her) was easier to use, while the students were unsatisfied with her choice of textbook. Her basic message is: It is important with material supporting all the requirements, but it is up to the teacher to connect teaching with learning strategies.

The actual use of these textbooks has not been studied. The textbook is a tool for learning mathematics, but should not be the only tool. The various demands on the teacher are huge, with respect to the knowledge to be taught and being able to help every student increase his or her individual knowledge base. I agree with the comment made by Wyndhamn, Riesbeck and Schoultz (2000) on the function of the textbook:

> A textbook becomes the pilot of a lesson if you accept a package without critically examining where you stand from a didactical point of view. (p.214, my translation)

For a teacher it is therefore important to have a clear standpoint on what teaching and learning are and to use material needed to maintain this. Knowing the didactical point of view of the textbook and trying to match this with one's own ideas on teaching and learning is important.

6.4 Suggestions for further work

During the analysis many questions arose that could not be studied further, mainly because of the limitations of this work in time and scope. A work like this is usually said to result in more questions than answers, and this study is no exception. Here, some of the questions of interest for me will be presented. Some questions will remain for the future or for others to study, while others will be used in my future research.

6.4.1 Use of material

This study is based on the analysis of mathematics textbooks in school year 7. By analysing the textbooks for school years 7, 8 and 9, a general picture can be presented on how the tasks are differentiated, since the result is based on a higher number of tasks.

Because only one specific chapter (on fractions) has been studied, it would be interesting to compare the results with a chapter with different contents. For example, are there any differences in the tasks when studying a chapter on fractions and a chapter on geometry?

Because the focus is on tasks in different strands, the assessment tasks used in the diagnostic test and the final test were not examined. These tasks have an important function in the textbooks, since they decide if the students have passed the course or not. Another interesting question is if the tests in the textbooks are comparable with the national test.

Studying the differences and similarities between countries that have had similar discussions concerning mathematics for all, changing from ability grouping to mixed ability grouping, could be a suggestion for further studies as well. For example, this could be studied in Scandinavia, England and other countries. The cultural aspects should then be in focus, since not all countries have the same curriculum.

6.4.2 The analysing tool

The performed analysis can be done in many ways and with many aspects in focus. When the tool was constructed , the aspects were those in connection to my questions. For the last two aspects (processes and demands), one implication of the study that was not done (but could be done) is to study the correlations between them. As their separate results show, their categories have clear connections. This is also discussed in the method chapter (see section 4.3.3).

Due to time limitations, an aspect regarding the use of text was omitted in this analysis. If the framework were to be revised, this aspect would be one natural addition. The text aspect has multiple categories because one can study, for example, the amount of text, the words in use and the context of the text. For more information on the analysis of tasks, see for example Selander (1995), Skyum-Nielsen (1995) and Österholm (2004). As the tasks were being examined, it was noticed that tasks at low-level strands contain fewer sentences and fewer words in the text. In strands at a higher level, the text seemed to increase. A study on the mathematical content, with a specific focus on the mathematical concepts, could also be done.

6.4.3 Connections to the classroom

A theoretical link between the theories of learning and differentiation would be of utmost interest. This link has not been found but is of interests to me in my future work and has therefore not been discussed in this work.

To understand what is going on in the Swedish classroom, more classroom research is needed. My central question is (and has been for a long time) the function of the textbook in mathematics education, i.e. use of textbooks by teachers and students inside and outside the classroom. As presented earlier, a textbook can by its content be analysed differently from its use in the classroom. One could suspect that this depends on the teacher and the student, as well as the content and structure of the textbook. Henningsen and Stuen (1997) have constructed a framework and identified three phases. Accordingly the first phase has been studied: tasks appearance in instructional materials. The continued phases for the differentiated tasks are the teachers' use and finally the implementation done by students. These phases are of interest for the continuation, when analysing the function of the textbooks in the classrooms. The study to date will therefore be a good introduction to my continuing work.

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A

A concept map

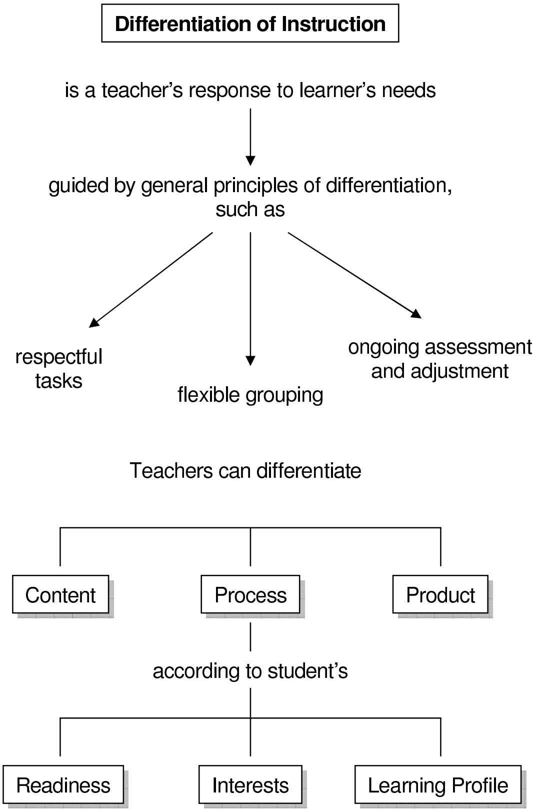

Figure A.1: A concept map for differentiating instruction (Tomlinson & Demirsky Allan, 2000, p.3)

B

Blooms taxonomy - A revised version

The Knowledge Dimension	The Cognitive Process Dimension					
	1. Remember	2. Understand	3. Apply	4. Analyse	5. Evaluate	6. Create
A. Factual Knowledge						
B. Conceptual Knowledge						
C. Procedural Knowledge						
D. Metacognitive Knowledge						

Table B.1: Blooms taxonomy table (Anderson & Krathwohl, 2001)

C

Diagrams ordered by aspects

Here follows the results of my analysis in form of the diagrams for each aspect.

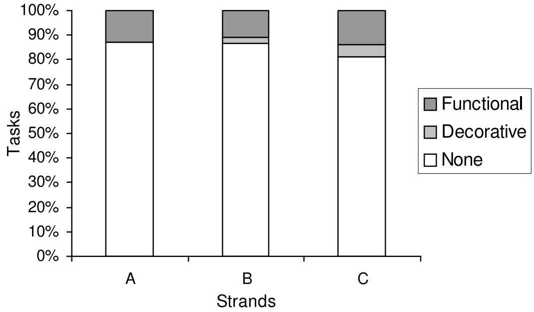

Figure C.1: Pictures in Matematikboken X, basic course

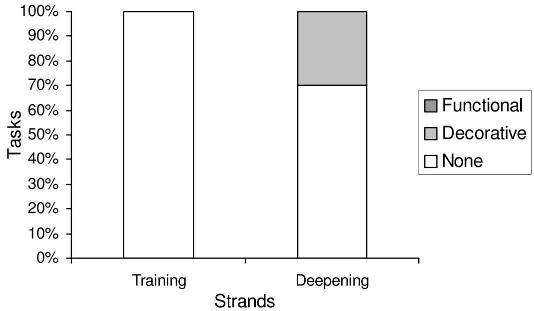

Figure C.2: Pictures in Matematikboken X, follow-up course

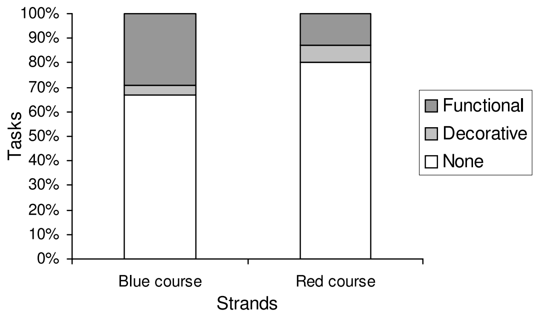

Figure C.3: Pictures in Matte Direkt 7

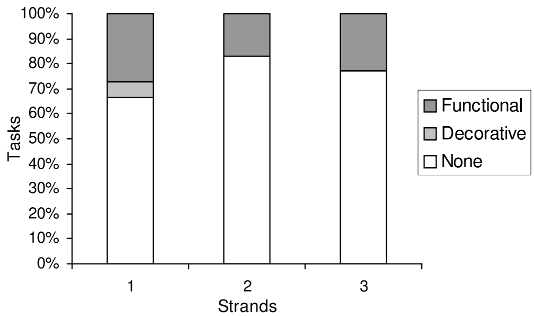

Figure C.4: Pictures in Tetra A

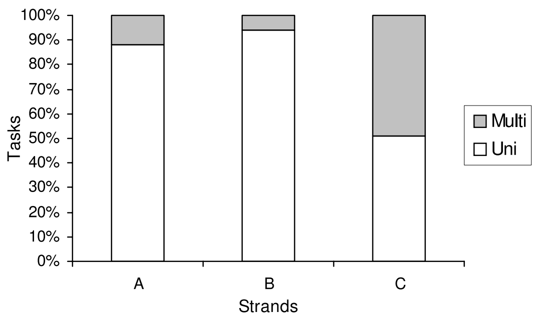

Figure C.5: Operations in Matematikboken X, basic course

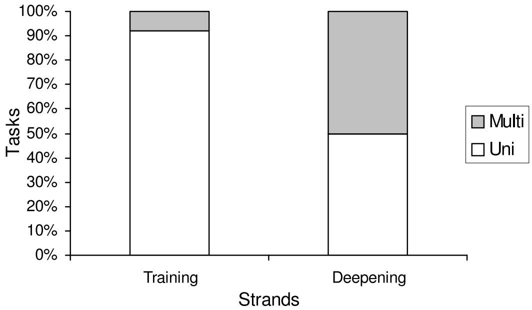

Figure C.6: Operations in Matematikboken X, follow-up course

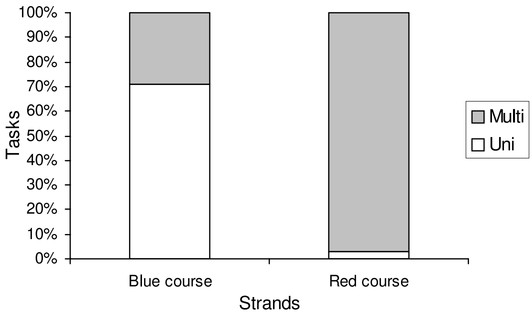

Figure C.7: Operations in Matte Direkt 7

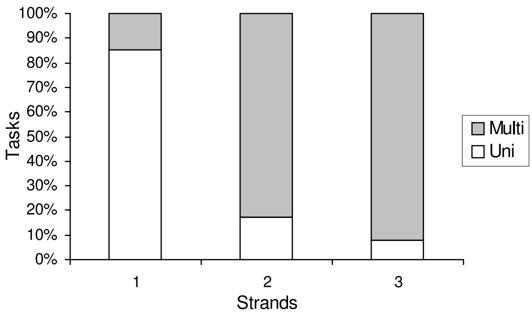

Figure C.8: Operations in Tetra A

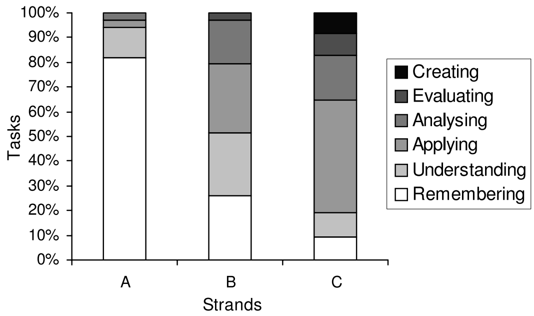

Figure C.9: Cognitive processes in Matematikboken X, basic course

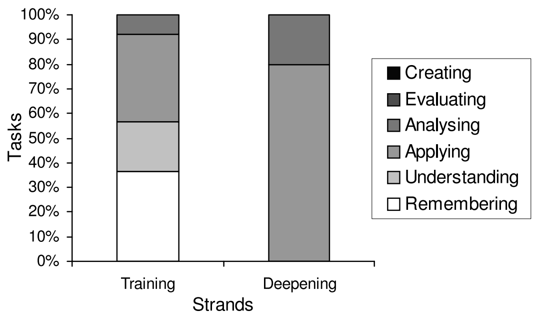

Figure C.10: Cognitive processes in Matematikboken X, follow-up course

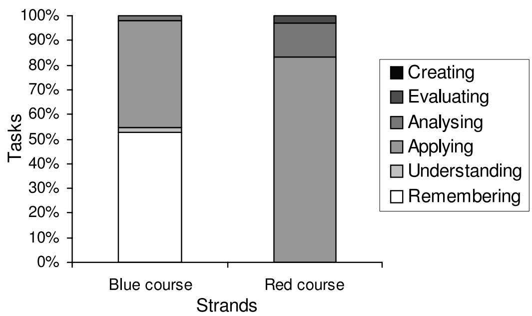

Figure C.11: Cognitive processes in Matte Direkt 7

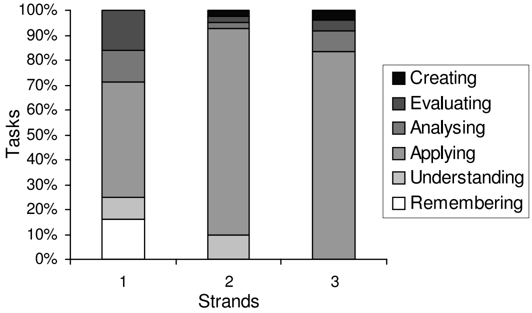

Figure C.12: Cognitive processes in Tetra A

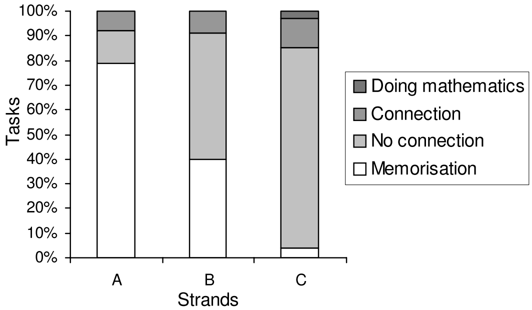

Figure C.13: Level of cognitive demands in Matematikboken X, basic course

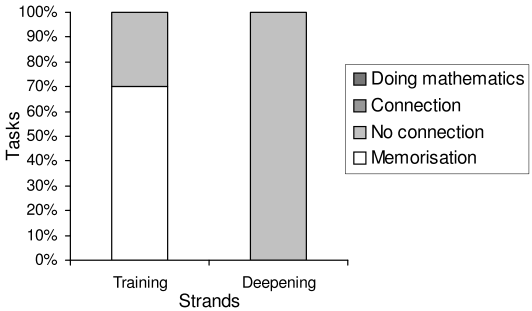

Figure C.14: Level of cognitive demands in Matematikboken X, follow-up course

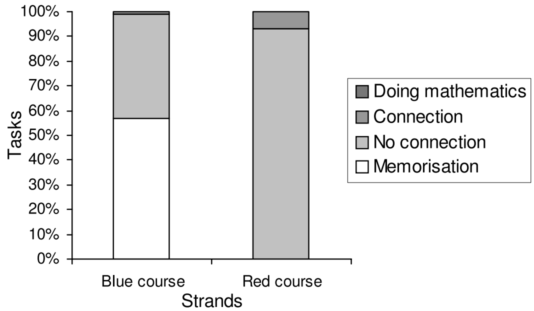

Figure C.15: Level of cognitive demands in Matte Direkt 7

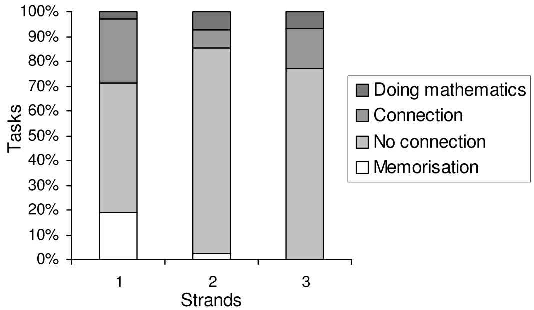

Figure C.16: Level of cognitive demands in Tetra A

D

Intercoder reliability

Use of pictures
$C_{1,2} = 366$
$C_1 + C_2 = 192 + 192 = 384$
$R = 0.95$

Required operations
$C_{1,2} = 370$
$C_1 + C_2 = 192 + 192 = 384$
$R = 0.96$

Cognitive processes
$C_{1,2} = 350$
$C_1 + C_2 = 192 + 192 = 384$
$R = 0.91$

Level of cognitive demands
$C_{1,2} = 358$
$C_1 + C_2 = 192 + 192 = 384$
$R = 0.93$